

FREQUENCY DOMAIN STRUCTURAL OPTIMIZATION
OF A FLEXIBLE MANIPULATOR INCORPORATING A PD
CONTROLLER IN THE DESIGN STAGE

By

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Abstract of Dissertation Presented to the Graduate School
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The study reported in this thesis investigates the potential advantages of integrated design strategies for flexible manipulators that utilize a controller in the process of optimizing the structural design.

A clamped free beam model with torque inputs at the joints and disturbances at the link ends is considered in the investigation. A clamped mass beam model is also investigated. A proportional and derivative controller is utilized in the design process with several objective functions that incorporate weighted combinations of the mass of the structure, modal mass, modal stiffness, and the natural frequency of the rigid and flexible modes. The results show that the objective function, which minimizes mass and maximizes the product of the modal mass and the modal stiffness with

appropriate weighting factors, gave the best dynamic performance. It was also shown, as would be expected, that the selection of the weighting factors is an important consideration since it can significantly influence the dynamic characteristics of the optimized system.

Although a two link tubular planar manipulator with a PD controller and assumed beam models for the links is considered in the optimization based on modal parameters, the reported approach can be extended to time domain optimization of multi-link spacial robots with complex structures designed to perform particular tasks.

CHAPTER 1 INTRODUCTION

The structural designs of existing industrial robots incorporate direct methods to minimize system vibration through relatively stiff links and drives. This design approach provides good static performance but also makes the robot massive. Massiveness lowers the system dynamic performance and increases power consumption. In some applications a light-weight robot is more attractive despite its reduced stiffness. Robots in space applications require light, long arms to be controlled with low energy in a wide operating range and good performance. Nuclear plants require light, long arms for access to areas of high radiation. In small products applications, the electric circuit chip mounter needs a high-speed manipulator making a light structure necessary.

The light-weight manipulator has many advantages such as low power consumption, a smaller actuator, more compact link design, lower system inertia, and less mass to be transported. Therefore, it can benefit from an improvement in rigid mode dynamics. The principal disadvantage is reduced accuracy in end-point control. Due to this accuracy drawback and the complexity of the modeling, light robots received little attention until the end of the 1970s. Since their use in a space manipulator,

considerable research has been performed particularly in the control area. An historical review¹⁻³ has provided general information on the modeling of the flexible manipulator, manipulator parametric studies, some of the design approaches and efficient control algorithms, etc.

The general approach for the design of manipulators is based on kinematics, stress analysis, and structural analysis. The classical design leads to an optimal structure from the mechanical viewpoint. Optimization of the system through the statics and the dynamics of the mechanical system does not guarantee that the manipulator system is optimal within a control loop. The total system overview and the structural design of the clamped body including controller effects have received attention in aerospace research since the middle of the 1980s. Manipulator designs that include controller effects began in the early 1990s. The work proposed in this study will treat the problem by optimizing the system for minimum vibration from the structural design viewpoint. The manipulator design further encompasses the rigid mode controller effects at the design stage.

Most existing robots are controlled by means of independent joint controllers based on classical control concepts such as PID control, robust control, etc. Vibration control of industrial robots and space manipulators is achieved by moving joints slowly and waiting until the end effector settles down. If the robot is stiff enough, the deflection can be neglected and a controller based on the rigid mode is usually adequate. In light-weight robots, deflection is not negligible, but there are several methods to

solve the problem. An additional active controller may quickly reduce structural deflection. A slow controller may avoid exciting the flexible mode. Manipulator flexibility can go inside or outside of a control loop. When flexibility is inside a control loop, then the stability of the system requires careful checking. This study includes the flexibility of the manipulator in the design process.

A simple controller is developed based on general control and optimization theory. The proportional and derivative controller is used due to its simple structure and the structural benefits it can provide in the design stage. This controller is used in the design stage to modify the manipulator dimensions, optimizing them based on structural considerations. The resulting system with controller effects is expected to undergo minimum vibration in both the rigid and the flexible body modes.

The research direction of this study is first to model the flexible manipulator with the assumed mode method and to investigate further the structural characteristics of the manipulator through various boundary and system input conditions. These structural parameters then are used to achieve an optimal structural design. This study focuses on an integrated design approach where a controller is used in the design stage to improve system performance inside a control loop.

CHAPTER 2 LITERATURE REVIEW

Modeling

The modeling of the manipulator is the first step in the design and control of a system. Proper modeling of a system is necessary for predicting performance characteristics as influenced by design control parameters. The robot structure can be divided into several components such as joints, drives, and links. These can be treated as rigid structures or flexible structures depending on the relative effects of the flexibility. Researchers usually choose a model that best fits their objective. Potential models for the manipulator can be categorized in the following way:

- (a) Rigid joint and rigid link model
- (b) Flexible joint and rigid link model
- (c) Rigid joint and flexible link model
- (d) Flexible joint and flexible link model

Most industrial researchers use the model that has rigid joints and rigid links. The links and the joints in most applications are designed to be as massive as possible in order to increase stiffness. The merits of this type of manipulator are the simple dynamics of the system and good position accuracy so that open loop control schemes are possible. The disadvantage

is the low level of system responsiveness to rapid tasks. Numerous studies have been done on the design, control, sensing, and vision of rigid robot systems.⁴ Industrial robot researchers are interested in modeling and control from a practical standpoint. The exact model of a manipulator is often complex and requires high-powered computing to perform real time estimation. That is why the simple model based on the rigid mode is preferred. The effects of the unmodeled characteristics in the simple model could be reduced by robust control schemes.⁵

A more realistic model of the industrial robot is the flexible joint and rigid link model. The sources of joint compliance are shafts, gear trains, harmonic drives, chain drives and others. Considerable research has been undertaken to study the effects of joint compliance. Sweet and Good⁶ and Good et al.⁷ showed that flexibility of the drive system is the dominant source of compliance in the industrial robot, GE-P50. Their fourth-order dynamic model was derived through experiments. The model shows resonances in the 5 - 8 Hz range because of flexibility in the harmonic drives, which is not considered in conventional rigid mode robot models. The nonlinear vibratory response of a two link robot manipulator with compliance in its joints has also been investigated by other researchers.⁸⁻⁹

Link compliance is significant in manipulators with slender links as in the case of the space shuttle remote robot. One of the pioneering research on the flexible link manipulator was done by Book et al.¹⁰ who used a

planar, nonlinear model with modal representation of flexibility to analyze the performance of control schemes. Smart and Wiens¹¹ investigated gravitational effects on the natural frequencies and the static deflections of two link flexible system with clamped mass model. Barbieri and Ozguner¹² investigated mode shapes for a flexible slewing link. Constrained and unconstrained mode expansions were investigated by changing system inertia in the absence of any controller dynamics. Oakley and Cannon¹³ presented equations of motion for a planar two link configuration by using Kane's dynamic equations. A novel approach for representing the mode shapes of the manipulator was presented with a system modes representation.

The most complex model for a manipulator is the flexible joint and flexible link. This model is the most realistic but is seldom used in design and control because of the complexity of its system equations.⁷ Ding et al.¹⁴ derived a general dynamic model for a large class of flexible robot arms incorporating joint and link elasticity. Some control schemes were based on this model and investigated via simulation,¹⁵ but the controllers employing joint and link flexibility can not meet the computational requirements for real-time implementation.

Design

The industrial sector has undertaken considerable research into mechanical designs of robots.¹⁶ Traditional design approaches add material

to increase the manipulator's stiffness and the strength. This conservative approach can improve positioning accuracy, but large mass and inertia result in reduced dynamic performance. The demand for fast, accurate manipulators requires designs of structurally advanced manipulators.

A general review has surveyed designs of light weight manipulators.¹⁻² The paper by Book et al.¹⁷ presents general strategies on solving design and control problems of light-weight arms. They divided problems of light manipulators into four categories: material and shape, feedback control algorithms, strategies of arm use, and arm trajectories. Their method to improve structural damping uses viscoelastic material and a length-optimized constraining layer.

One approach for an advanced structure uses composite materials. Dunbar et al.¹⁸ introduced the use of composite material for designing booms in the space shuttle's remote manipulator system. Ultra-high modulus graphite/epoxy was chosen to meet stiffness requirements within weight constraints. A paper by Hunter et al.¹⁹ describes the design considerations for the shuttle's remote manipulator system. The total weight of the manipulator, the maximum diameter of the arm booms, and the stiffness of the arm booms are discussed as constraints. Saravanas and Lamancusa²⁰ proposed a design method for predicting the optimum configuration of the ply angle and ply thickness in serial links. This design goal simultaneously maximizes the end effector stiffness and payload carrying capacity of composite robotic manipulators through changes in ply

configuration. Their design methodology is applied to a SCARA-class manipulator for illustration of its effectiveness.

Model order reduction is an interesting approach to shorten the structural design process. Paz²¹ introduced an algorithm for modal condensation that transforms systems with a high degree of freedom into systems with a relatively low degree of freedom through checking residual energy for perturbations. Equations producing little energy are neglected because of their lack of effect on the entire structure. Kim²² applied the modal condensation method to the structural redesign of a uniform curved shell segment. The thicknesses of the shell elements are redesigned to minimize the vibration of a certain mode at a particular point.

When a system is more complex, a systematic method is needed for design. Sobieski et al.²³ structurally designed three-beam elements through multilevel optimization, a method which partitions a structure into a number of substructuring levels. A three-beam structure is divided into three levels: a bottom level representing the plates, a middle level for the beams, and a top level for the assembled structure. Each level has different objective functions, design variables, and constraints.

Some researchers include control theory into the design stage to improve performance. Jaliwala et al.²⁴ presented the idea of using control theory to optimize a machine element. A cam follower mechanism is modeled as a closed loop control system. The cam shape is designed by

minimizing the error between the ideal and the actual follower lift. Haftka et al.²⁵ performed a structural modification for a cruciform beam that was supported by cables and was controlled by several rate-feedback collocated sensor-actuator pairs. Active damping in the flexible modes is created by the rate-feedback controller rather than structural damping. The sensitivity of the active damping gains to minor structural modifications are calculated; the thicknesses of highly sensitive elements consequently are changed. The control efforts are reduced due to the low gains in the flexible mode controller by changing the masses of sensitive elements.

The dynamics of the structure can be altered through the dynamics of the controller. Interactions between controller and structure are receiving attention in an effort to improve total system performance. Spanos²⁶ investigated control-structure interaction problems via stability analysis of a linear servo-loop model. Structural flexibility is categorized into three types based on the magnitude and the location of zero in the s-plane. The stability of these systems is investigated by changing the relations between the closed loop bandwidth and the modal frequency of the structure. Sensors, filters, and PD controller are included to change the closed loop bandwidth. Dyke et al.²⁷ investigated control-structure interactions and performed examples of including control-structure interaction in protective system design. Hydraulic actuators are modeled in three different ways and applied to the prototype with three active control systems. A natural velocity feedback link is modeled to represent the interaction between the

hydraulic actuators and the structures. Their examples showed better system performances by accounting for control-structure interaction and actuator dynamics.

A study of control-structure interaction resulted in a different approach for the design of the manipulators. Park and Asada²⁸⁻²⁹ presented a new transmission mechanism for a single link flexible arm. The endpoint positioning of a flexible arm produces a non-minimum phase system, which is difficult to stabilize. The proposed torque transmission mechanism transformed the single link flexible arm to a minimum phase system so that control is easier than in regular flexible arms. The unstable zeros of the control system are removed by relocating the torque actuation point.

A number of studies on the integration of structure and control design were conducted from the middle 1980s to the early 1990s, particularly concerning large space structures. Most of the mechanisms investigated in this area were clamped structures and the flexible modes are controlled by time domain control schemes. LQ-based control is the dominant control scheme with the feedback of flexible modes because it fits well with the structure of the objective function for the optimization problems. Khot et al.³⁰ proposed an algorithm to design a minimum-weight structure with a constraint on the damping parameter of the closed loop system. Closed loop system equations of large flexible structures are calculated with the feedback of flexible mode responses. A controller employing a linear quadratic regulator is utilized; the performance index

for the controller is chosen to find the optimum gain matrix through a compromise between minimum error and minimum energy criteria. After a closed loop control system is completed, the gradients of the damping factors are checked by changing the cross-sectional area and a minimum-weight structure is found within the damping constraints. McIntosh and Floyd³¹ have developed a technique for a least-weight optimal design of a tubular-truss space structure with constraints on its natural frequencies and its open loop disturbance rejection properties. The disturbance rejection properties of the structure are measured by a special function, disturbance-to-regulated-variable grammians. Kosut et al.³² showed another strategy for vibration suppression due to disturbances. The structural parameters for disturbance attenuation are chosen to reduce the necessary control authority. The ratio of two transfer functions (the output over the disturbance and the output over the input torque) is used to seek the minimum objective function value.

The integrated structure and control design is extended with different strategies. Bodden and Junkins³³ proposed a group eigenvalue optimization approach for a high-order system by using structure and controller design iterations. The modal matrix that includes eigenvalues and eigenvectors is used for the structural and the control optimality criteria. Exact eigenvalue placement or pole placement is not a reasonable design approach for a high-order system; as a consequence they undertook

a more reasonable approach that moves all of the eigenvalues into an acceptable region of the complex plane. Other researchers have extended the integrated optimization approach. Lim and Junkins³⁴ included a new design algorithm for numerical applications incorporating stability robustness measures. They compared the robustness of optimization schemes through stability robustness and eigenvalue sensitivity. The results indicate that maximizing a stability robustness measure produces more robust designs than directly minimizing the eigenvalue sensitivity. Three different cost functions (total mass, stability robustness, and eigenvalue sensitivity) are optimized with respect to a set of design parameters that include the structural and control parameters and actuator locations. Miller and Shim³⁵ presented a simultaneous structural and control optimization. The objective function is a combination of mass and total system strain and of kinetic and control energies with an appropriate weighting factor. Ten bar truss is adopted for a numerical example with linear quadratic control. The objective function is calculated through a gradient-based search.

Considerable research exists on the integrated structure and control design of space structures. Gilbert and Schmidt³⁶ presented general information on multilevel structure and control design. They have proposed a new approach as an alternative to both the simultaneous and the sequential integrated control and structure design methods. The structure

and the control laws are designed independently and integration of the two properties is achieved through a special optimization formulation so that their method utilizes multilevel optimization.

Another integrated approach is structural tailoring. Belvin and Park³⁷ proposed a method for optimization of a closed loop structural system using structural tailoring. This paper described a choice of weighting matrices based on physical considerations and an approximate cost function that minimizes the control energy needed to suppress vibrations in a clamped structure. The tailoring procedure is based on the open loop eigenvalues and eigenvectors, and a weighting matrix is chosen in a way that the low frequency modes contribute most to the cost index.

Robot arms with integrated structure and control design have received attention since the early 1990s for a single link flexible manipulator. Asada et al.³⁸ investigated the dynamic characteristics, particularly pole-zero location, in relation to design parameters, i.e., the shape and structure of the arm mechanism. This strategy formulates the inverse relationship of the pole-zero plot back to the arm structure using a generalized Jacobian method. Asada and coworkers solved the inverse problem of finding an optimal structure that satisfies the desired pole-zero specifications for a single link flexible arm. Rai and Asada³⁹ applied this concept to a flexible manipulator by using singular value decomposition. Singular value was interpreted as a scale factor between the vectors in the

design variables space and the vectors in the system characteristics space. Jacobian matrix mapping between the design space and system characteristics space were calculated; the magnitude of the singular value of the Jacobian matrix served as a criterion for the trade-off analysis between the conflicting properties. Their research is expanded with the same Jacobian matrix by including the travel time for a given task.⁴⁰ They investigated solutions to time-optimal control problems with respect to the arm link inertia and structural flexibility. The single arm structure was redesigned to minimize traveling time through trade-off analysis between arm inertia and its natural frequency.

Very few studies are published on integrated structure and control design of a two link flexible manipulator. Park and Asada⁴¹ introduced the integrated design for high-speed positioning. Control gains for the rigid modes were designed first based on maximizing the dominant rigid mode pole by changing the actuator location. The design parameters were arm geometry, including length, width and height, with system mass and length constraints. The parameters were optimized to maximize the real part of the rigid mode dominant pole that represents the speed of response.

Objective of This Study

A literature review revealed that design of a two link flexible manipulator remains a challenging research area, particularly in considering the controller effects.

This study will design a two link manipulator by using modal parameters. Consequently, the structural characteristics of the two link manipulator are investigated within given design variable constraints. The clamped free and clamped mass models are analyzed and compared. The effects of general input locations on modal stiffness and modal mass are also investigated.

After the structural characteristics are investigated, the optimal structural design is performed based on the modal parameters. This study introduces three structural optimization schemes that are based on natural frequency, modal stiffness, and the product of the modal mass and the modal stiffness, respectively. All of these schemes tend to make a system so massive that a conflicting property, mass, is included to find an optimal structure within diametric constraints.

After structural optimization is accomplished, the integrated structure and control design of the flexible manipulator is undertaken. The structural design itself does not guarantee that the designed manipulator is optimal when it is included in the control loop. A PD controller is designed for the structurally optimal two link flexible manipulator. A new approach is introduced to include controller effects by using rigid mode modal parameters. The design of the final system includes the effects of the modal parameters of both the rigid and the flexible modes. The approach followed in this research, which focuses on the integration of the

structure and control design, shows that better system performance is achievable when the controller is included in the design stage.

CHAPTER 3 SYSTEM MODELING

Introduction

System modeling is the first step in machine design and control. Inadequate modeling can lower system performance. Overly accurate modeling, however, can result in excessive complexity that makes analysis intractable and control extremely difficult. The model need not simulate all physical characteristics when it is used in the design stage. Only the important characteristics should be included in the model and some engineering judgement can be relied upon to find an optimal system without losing the main features of the system. Accordingly, most researchers use linearized and reduced-order models in order to simplify system analysis and control. In this chapter, the complex system equations will be simplified by using linearizing schemes in treating equivalent beam model, vibration modes, and boundary conditions.

Another factor influencing the type and accuracy of the model is the control scheme. Unmodeled mechanical properties such as friction or damping can cause deterioration in system performance especially when an open loop control scheme is used. For example, the dynamic model inversion technique requires exact modeling because the inverse of the

transfer function is used to cancel unwanted system characteristics without considering disturbances. The performance of an LQ-based controller is more dependent on the system model because of sensitivity to weighting factors. These kinds of control schemes need exact modeling if controller robustness is to be assured. When a robust control scheme is used such as PID type controller or adaptive controller, the system performance is less sensitive to disturbances so that exact and complex modeling is not necessary.

The modeling of flexible manipulators has been studied by many researchers during the last three decades. The design and control of flexible manipulators have attracted attention since their use in space shuttles. Many approaches have been proposed for modeling of the flexible manipulator and can be grouped into three categories: PDE (partial differential equation), FEM (finite element method), and AMM (assumed mode method). Partial differential equations⁴² can yield precise mathematical models but may cause formulational and computational difficulties and consequently most application of this method is limited to simple structures. One of the recent popular methods is the finite element methods that utilizes a set of linear ordinary differential equations. This method can derive a detailed mathematical model for the structure and allow flexibility for modeling arbitrary cross sections and arbitrary boundary conditions.⁴³ The complexity of this type of modeling makes difficult its application in a design stage that includes controller effects.

The finite element model also produces high frequency elastic modes and the amount of computations can be so extensive that the numerical solution of the system equations is difficult. The third approach is the assumed mode method in which partial differential equations are simplified through transformation into ordinary differential equations. This is the most commonly used modeling method for the controller design of a manipulator because a relatively small number of states can be used to approximate a distributed parameter system. This approach avoids the high frequency problems and is implemented easily using computers. The limited modeling flexibility of this method makes its application difficult for systems with complex shapes, but it is an attractive method for manipulator modeling of a generally simple structure and long arms. This method is adopted in this study for modeling the flexible modes of the manipulator and including the controller effects in the design process.

There are two types of models for the beam: the Timoshenko beam and the Euler-Bernoulli beam. The Timoshenko beam includes most of the effects that might take place, e.g., shear, bending deformation, and rotary inertia effects. Shear deformation is insignificant in the planar motion robot and axial displacement is also negligible due to insignificant axial forces. The bending deformation of the beam is the only effective and considerable deformation in this study. Accordingly, the Euler-Bernoulli beam model is adopted to determine the flexible modes of the manipulator.

The general modeling method developed by Book et al.¹⁰ is adopted for this study.

This chapter presents the application of the Euler-Bernoulli beam model and the assumed mode method to the flexible modes of the manipulator. The system energies, kinetic and potential energy, will be derived to obtain the system equations via the Lagrange equation. The resulting system is described by 6 DOF nonlinear equations with 2 DOF for the rigid modes and 4 DOF for the flexible modes. This nonlinear system model is then simplified based on linearizing schemes for the design and control process. The system analyzed by the assumed mode method will be compared with that constructed by FEM.

Model Description

The model considered in this study has two links and incorporates rigid and flexible modes for each link. The flexibility of each link is expressed as an Euler-Bernoulli beam model in which lateral vibration is the only dominant vibration characteristic. The two joints are assumed to be frictionless with no backlash. The flexibility of the joints is also assumed to be negligible compared to that of the long arms. The condition of the joints and the dynamics of the controller may change the boundary conditions and dynamics of the mechanical system. Consequently, two different models, clamped free and clamped mass, will be considered. The

effects of the gravity will not be considered and, therefore, the equivalent stiffness due to gravity is not included in the rigid body modes.

The real motion of the beam is a coupling of the rigid body motion and the elastic deformation. To describe the multibody dynamics including the flexible modes, two types of coordinate systems are used: the global (fixed) coordinate system and the local (moving) coordinate system. The rigid body motion can be represented by the coordinate transformations between two adjacent links and the elastic motion is expressed in terms of the deflection of each link with respect to the local coordinate. A schematic diagram of the system is shown in Figure 3-1.

The links are considered to be hollow cylinders and the ratio between the external diameter and the thickness is fixed at 0.02. The selection of this ratio and the shape of the cross section are guided by consideration of applicability of beam theory for plane sections to remain plane. The diameters of the two links will be considered as the design parameters influencing the cross-sectional area, the stiffness, and the mass distribution of the manipulator system. The slopes at the end of each link are included by using the assumed mode method in which these are the first partial derivative of the link deflection. The end effector is represented by a payload which has mass and rotational inertia.

Referring to Figure 3-1, the axes of coordinates are described with the notation of $[\hat{x}_i, \hat{y}_i]$ and the subscript 'i' is 0 for the fixed coordinate

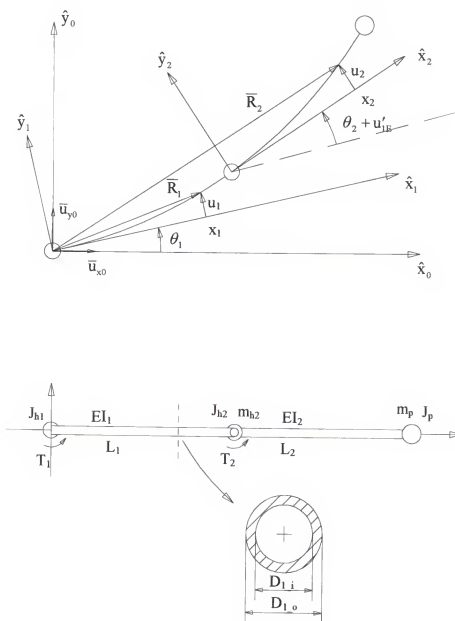


Figure 3-1 Schematic diagram of the arm
 (a) Coordinate systems
 (b) Nomenclature of the elastic arm model

and 1, 2 for the moving coordinates on the first and second links respectively. The unit vectors on the frame $[\hat{x}_i, \hat{y}_i]$ are defined as

$$\begin{bmatrix} \bar{u}_{x_i} \\ \bar{u}_{y_i} \end{bmatrix} \quad (3-1)$$

and the deflections on each link are u_1 and u_2 respectively. The subscript 'E' will be used to represent the end of the link; i.e., u_{1E} represents the deflection at the end of the first link. A position on the flexible link can be expressed by the summation of two perpendicular position vectors as

$$\bar{x}_i + \bar{u}_i = x_i \times \bar{u}_{x_i} + u_i \times \bar{u}_{y_i}. \quad (3-2)$$

The slope at the end of each link is defined as

$$u'_{iE} = \left. \frac{\partial u_i}{\partial x_i} \right|_{x_i=L_i} \quad (3-3)$$

and transformation matrix and angular velocity matrix are

$$[TR_i] = \begin{bmatrix} \cos \theta_i^* & -\sin \theta_i^* \\ \sin \theta_i^* & \cos \theta_i^* \end{bmatrix} \quad (3-4)$$

$$[\Omega_i^*] = \begin{bmatrix} 0 & -\dot{\theta}_i^* \\ \dot{\theta}_i^* & 0 \end{bmatrix} \quad (3-5)$$

where $\theta_1^* = \theta_1$ and $\theta_2^* = \theta_1 + \theta_2 + u'_{1E}$.

By using the above notations, the position vectors of any point in beams 1 and 2 are represented as follows

$$\begin{aligned} \bar{R}_1 &= [TR_1] (\bar{x}_1 + \bar{u}_1) \\ \bar{R}_2 &= \bar{R}_{1E} + [TR_2] (\bar{x}_2 + \bar{u}_2) \end{aligned} \quad (3-6)$$

The velocity vectors can be obtained by differentiating the position vectors with respect to time and expressed as

$$\begin{aligned}\bar{V}_1 &= [TR_1] [\dot{\Omega}_1^*](\bar{x}_1 + \bar{u}_1) + [TR_1](\dot{\bar{u}}_1) \\ \bar{V}_2 &= \bar{V}_{1E} + [TR_2] [\dot{\Omega}_2^*](\bar{x}_2 + \bar{u}_2) + [TR_2](\dot{\bar{u}}_2).\end{aligned}\quad (3-7)$$

These vector notations, which are similar to those used by Meirovitch and Chen,⁴⁴ will be used to calculate the kinetic and the potential energy of the Lagrange equation and to formulate the system equations of the two link manipulator.

Assumed Mode Method

This section presents the assumed mode method and how to derive mode shapes for the flexible modes. The governing equation for the deflection of the beam is the Euler-Bernoulli equation which can be written as

$$EI \frac{\partial^4 u^1(x,t)}{\partial x^4} + m \frac{\partial^2 u^2(x,t)}{\partial t^2} = 0 \quad (3-8)$$

where m is the mass per unit length, EI is the flexural rigidity. The deflection is a function of position and time as shown in the fourth order partial differential equation. Direct use of the PDE is difficult to solve for the deflection. Consequently, a simpler procedure is considered, the so called separation of variables. This method leads to convenient expressions of the deflection by changing the partial differential equations into ordinary differential equations to derive the generalized mass and stiffness. Accordingly, the deflection can be expressed by a linear

combination of admissible function $\phi(x)$ multiplied by time dependent general coordinate $q(t)$ as

$$u(x, t) = \phi(x) q(t). \quad (3-9)$$

The admissible functions are arbitrary functions that satisfy the geometric boundary conditions and need not satisfy all natural boundary conditions.

By substituting equation (3-9) into equation (3-8), the original equation is divided into two ordinary differential equations as

$$\begin{aligned} \frac{d^4 \phi(x)}{dx^4} - \beta^4 \phi(x) &= 0 \\ \frac{d^2 q(t)}{dt^2} + \omega^2 q(t) &= 0 \end{aligned} \quad (3-10)$$

where $\beta^4 = m\omega^2 / EI$, and ω is the vibration frequency of the beam. The solution of the first equation is the mode shape of the beam and that of the second equation is the time dependent harmonic response with a certain frequency.

When the mode shapes are calculated, boundary condition problems are encountered. Most researchers use a clamped free beam model for the single link flexible manipulator wherein it is straightforward to calculate the modes. The adoption of the clamped free beam model implies zero slopes for the deflections at the joints. When a multilink manipulator is modeled, each link is considered to be an independent beam clamped at one end. Using the clamped modes as the expansion function is an approximation, and the exact mode is not defined. The reasons are that, first, the mode shape depends on the angle of the joints; second, the elastic

deformation at the end of the previous link contributes the angle of the next link; third, the controller also effects the dynamics of the joints. The system dynamics of the multilink manipulator are nonlinear and are dependent on configuration and controller. This study adopts both clamped free and clamped mass models for the two link manipulator as an approximation, although they do not accurately represent the real physical modes of the system.

The boundary conditions are divided into two categories, the geometric and the natural boundary conditions. The geometric boundary conditions are the deflection and the slope at the clamped side and can be written as

$$u(0,t) = 0 \quad \text{and} \quad \frac{\partial u(0,t)}{\partial x} = 0 \quad (3-11)$$

The natural boundary conditions are the moments and the shear forces at the end of the links. The shear force is used to distinguish the clamped free mode and the clamped mass mode because the clamped free mode has no shear forces at the end of the link. The natural boundary conditions for the clamped mass mode are

$$\begin{aligned} EI \frac{\partial^2 u(L,t)}{\partial x^2} &= 0 \\ EI \frac{\partial^3 u(L,t)}{\partial x^3} &= M_{\text{payload}} \frac{\partial^2 u(L,t)}{\partial t^2} \end{aligned} \quad (3-12)$$

By substituting the boundary conditions into the solution of equation (3-10) and equating the determinant of the coefficient matrix to zero, the frequency equation for the clamped mass mode can be derived as

$$1 + \cos(\lambda_r L) \times \cosh(\lambda_r L) - \frac{M_{\text{payload}}}{M_{\text{link}}} \times \lambda_r L \times (\cos(\lambda_r L) \times \sinh(\lambda_r L) - \sin(\lambda_r L) \cosh(\lambda_r L)) = 0 \quad (3-13)$$

where r is the number of the vibration mode and $\lambda_r L$ are the roots of the frequency equation. This equation is used to calculate the natural frequencies of the beam and larger payloads would lower the natural frequency of the beam. The third term of the left side of the equation is zero for the clamped free mode due to the no payload effect. Using four boundary conditions and the solution of frequency equations, the resulting mode shape of the single link can be expressed as

$$\phi_r(x) = (\cosh(\lambda_r x) - \cos(\lambda_r x)) - \frac{[\cosh(\lambda_r L) + \cos(\lambda_r L)]}{[\sinh(\lambda_r L) + \sin(\lambda_r L)]} (\sinh(\lambda_r x) - \sin(\lambda_r x)). \quad (3-14)$$

As we can see from equation (3-14), there are theoretically an infinite number of solutions for the frequency equations. In practice, it is not necessary to include all vibration modes to simulate all system characteristics. Since the response contributions of the higher modes of the flexible links decay fast and the amplitude of these modes are generally small, the deflection is dominated by the low frequency modes. Most researchers who use the constraint mode analysis include system vibration modes 1 or 2. In this study the number of the significant modes was

chosen to be two for each link. This should be enough to represent the main characteristics of the system for the design of the manipulator.

The deflections are replaced by product of mode shapes and coordinates based on the assumed mode method. Two modes of vibration are added for total deflection and the resulting deflections can be expressed as

$$\begin{aligned} u_1(x_1, t) &= \phi_{11}(x_1)q_{11}(t) + \phi_{12}(x_1)q_{12}(t) \\ u_2(x_2, t) &= \phi_{21}(x_2)q_{21}(t) + \phi_{22}(x_2)q_{22}(t) \end{aligned} \quad (3-15)$$

where ϕ_{11} , ϕ_{12} , ϕ_{21} and ϕ_{22} are the mode shape functions and q_{11} , q_{12} , q_{21} , and q_{22} are the generalized coordinates. The first number of the subscript represents the number of the link and the second the number of the vibration mode. The deflections in equation (3-15) will be used to calculate the kinetic and potential energies of the system in terms of the generalized coordinates.

Kinetic Energy

The kinetic energy is derived here to obtain the system equations by using the Lagrange method. The kinetic energy of the two link flexible manipulator is the summation of the kinetic energy of each component as

$$KE = \sum_{i=1}^2 T_{bi} + \sum_{i=1}^2 T_{Li} + T_p \quad (3-16)$$

where T_h , T_L , and T_p are the kinetic energies of the hub, link, and payload respectively. Each component of the system kinetic energy can be expressed as below

$$\begin{aligned}
 T_{hi} &= \frac{1}{2} \sum_{i=1}^2 J_{hi} \dot{\theta}_i^* + \frac{1}{2} m_{h2} \bar{V}_{1E} \cdot \bar{V}_{1E} \\
 T_{Li} &= \frac{1}{2} \sum_{i=1}^2 \left(\int_{m_i} \bar{V}_i \cdot \bar{V}_i dm \right) \\
 T_p &= \frac{1}{2} m_p \bar{V}_{2E} \cdot \bar{V}_{2E} + \frac{1}{2} J_p (\dot{\theta}_2^* + \dot{u}_{2E}')^2 .
 \end{aligned} \tag{3-17}$$

The kinetic energies are expressed in terms of velocity and mass. The velocities are composed of, first, the linear and angular components, second, the rigid and flexible mode components. The angular velocity includes the slope due to deflection at the end of the link.

Potential Energy and Virtual Work

The potential energy is due to the strain energy of the beam stored when it is bent. The gravitational effects on energy are not considered in the planar motion. The potential energies of each link are calculated separately to obtain the potential energy of the whole system. The potential energy for two mode shapes, clamped free and clamped mass, are calculated separately in numerical simulations. The deflection of beams is replaced by the admissible function multiplied by the general coordinates based on the assumed mode method. The potential energy of the two link manipulator can be expressed as

$$PE = \sum_{i=1}^2 \frac{1}{2} \int_0^{l_i} EI_i \left(\frac{\partial^2 u_i}{\partial x_i^2} \right)^2 dx \quad (3-18)$$

where E is the modulus of elasticity and I is the moment of the cross sectional area of each link. The final expressions for the kinetic and potential energy are given in Appendix A and will be transformed into the mass and stiffness matrix of the system by the Lagrange method.

As seen in equation (3-18), the potential energy of the manipulator depends on the length of the link, shape of the cross sectional area, material property of the beam, and payload from the structural viewpoint. The potential energy also depends on the acceleration of the beam, frequency components of input, and frequency characteristics of the controller from the control viewpoint.

Virtual work, which is the work done by all the outside forces in a virtual displacement, is used to obtain the external forces of the system in the Lagrange equation. For the manipulator, virtual work is defined by both torque in virtual angular displacements at the joints and disturbance force in virtual linear deflections at the end of each link. The form of virtual work can be expressed as

$$VW = \sum_{i=1}^2 (T_i \times \delta\theta_i^* + F_i \times \delta u_{iB}). \quad (3-19)$$

This equation implies that the virtual work has components of both the rigid and the flexible mode motions. The angular displacement due to the slope at the end of the previous beam is considered at the joints and the

linear deflection at the end of each link is also included in the flexible modes.

Damping

Damping can be categorized in two ways: viscous damping for the rigid mode and structural damping for the flexible mode. In the rigid body mode, damping is necessary to increase the stability of the system. However, it is not always practical to increase the damping structurally. One practical way is the use of the control concept. When velocity feedback is used, the effect of the viscous damping is not significant compared to the damping effect of the velocity feedback. This is one reason why an exact model of the system is not necessary in the design stage. The system used here will utilize velocity feedback control and viscous damping will not be included in the analysis.

In the flexible mode, however, structural damping is included. It is not easy to determine the structural damping theoretically. Many researchers use the proportional damping in which damping is proportional to the linear summation of the mass and stiffness, even though it is not applicable for all structural vibration problems. A more convenient way to evaluate the structural damping is the modal analysis in conjunction with experiments. Most machine members such as robot arms have damping ratios around 1-4 percent. This study assumes the structural damping ratio as 1 percent for both the first and second modes of the link.

Structural damping coefficients for the clamped beam model can be written as

$$c_{ij} = 2 \times \zeta \times \frac{k_{ij, \text{eff}}}{\omega_{n, ij}} \quad (3-20)$$

where ζ is the damping ratio, ω_n is natural frequency, k_{eff} is the effective stiffness for a certain vibration mode, and the subscript ij represents i -th link and j -th mode.

In order to include damping in the Lagrange equation, the Rayleigh dissipation function $\mathfrak{J}$ is used as

$$\mathfrak{J} = \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n c_{ij} \dot{q}_i \dot{q}_j \quad (3-21)$$

where n is the number of the generalized coordinates and $\dot{q}$ is the time derivative of the generalized coordinates. The velocity feedback will be used to determine the damped rigid mode response and the structural damping is to evaluate the flexible beam responses in time domain simulation.

System Equation

After expanding the vector notation energies derived in the previous sections, the Lagrange equation uses scalar quantities of kinetic energy and potential energy to derive system equations. Its general form can be expressed as

$$\frac{d}{dt} \left(\frac{\partial KE}{\partial \dot{q}_n} \right) - \frac{\partial KE}{\partial q_n} + \frac{\partial PE}{\partial q_n} + \frac{\partial \mathfrak{F}}{\partial \dot{q}_n} = Q_r \quad (n = 1, 2 \dots 6) \quad (3-22)$$

where the quantity Q_n is the generalized force corresponding to the selected generalized coordinates q_n , $\{\theta_1, \theta_2, q_{11}, q_{12}, q_{21}, q_{22}\}^T$ respectively. The generalized coordinates q_n show 2 rigid mode variables and 4 flexible mode variables so that the system has 6 DOF.

By substituting the derived scalar kinetic, potential and damping energy into the Lagrange equation, the resulting dynamic equation of motion can be expressed in the matrix form

$$[NM]\{\ddot{q}\} + [C]\{\dot{q}\} + [K]\{q\} + \{NF\} = [TC]\{T\} \quad (3-23)$$

where

$[NM]$: nonlinear mass matrix (6×6)

$[C]$: damping matrix (6×6)

$[K]$: stiffness matrix (6×6)

$\{NF\}$: nonlinear force (6×1)

$[TC]$: torque coefficient matrix (6×4)

$\{T\}$: external input (4×1), $\{T_1, T_2, F_1, F_2\}^T$

Equation (3-23) includes nonlinear terms in the mass matrix and the force vector. These nonlinearities complicate the design and control schemes and a linearized model will be derived for simplicity. The higher-order terms, such as coordinate coupling terms of the elastic motion, are neglected. The sines and cosines are replaced by the first terms of its Taylor series representation with a small angle assumption. The resulting linearized equation of motion is time-invariant and can be expressed as

$$[M]\{\ddot{q}\} + [C]\{\dot{q}\} + [K]\{q\} = [TC]\{T\} . \quad (3-24)$$

The elements of the matrices in equation (3-24) can be written as

$$[M] = \begin{bmatrix} M_{rr} & M_{rf} \\ M_{rf}^T & M_{ff} \end{bmatrix}$$

$$\text{diag}([C]) = \{ 0, 0, C_{11}, C_{12}, C_{21}, C_{22} \} \quad (3-25)$$

$$\text{diag}([K]) = \{ 0, 0, K_{11}, K_{12}, K_{21}, K_{22} \}$$

where the subscript ‘rr’ in the mass matrix means the rigid mode elements, ‘ff’ the flexible mode elements, and ‘rf’ the coupling elements of the rigid and the flexible mode. The main components of the mass matrix can be expressed as

$$M_{rr} = J_h + J_{\text{Link}} + J_{\text{payload}} + M_{\text{payload}} L^2$$

$$M_{rf} = m L^2 \int \phi(x) x dx + M_{\text{payload}} \phi_E + J_{\text{payload}} \phi'_E$$

$$\text{diag}(M_{ff}) = \int m \phi(x)^2 dx + M_{\text{payload}} \phi_E^2 + J_{\text{payload}} \phi'^2_E$$

$$\text{off-diag}(M_{ff}) = M_{\text{payload}} \phi_E \phi'_E + J_{\text{payload}} \phi'_E \phi'_E \quad (3-26)$$

This mass matrix is symmetric and should be positive-definite to represent a physically realizable system.⁴⁵ A necessary and sufficient condition for a matrix to be positive-definite is that all determinants obtained from a matrix should be positive. The elements of the stiffness matrix can be expressed as

$$K_{ij} = EI_i \int_0^L (\phi_{ij}(x))^2 dx . \quad (3-27)$$

The integration involving the mode shape functions are calculated by numerical method. The details of the components of the matrices [M], [C], [K] and [TC] are provided in Appendix B.

In order to include equation (3-24) in the control loop, it is convenient to use the state space form and the resulting equation can be written as

$$\{\dot{X}\} = [A]\{X\} + [B]\{T\} \quad (3-28)$$

where

$$\{X\}_{12 \times 1} = \{\{q\} \{\dot{q}\}\}^T \quad (3-29)$$

$$[A]_{12 \times 12} = \begin{bmatrix} 0 & I \\ -[M]^{-1}[K] & -[M]^{-1}[C] \end{bmatrix} \quad (3-30)$$

$$[B]_{12 \times 4} = \begin{bmatrix} 0 \\ [M]^{-1}[TC] \end{bmatrix}. \quad (3-31)$$

Model Comparison with FEM

The system model derived by the assumed mode method is compared to that obtained from the finite element method in this section. In general any assumed model should be verified with a different model or experiment to check the reliability of the modeling method. A finite element analysis program is used to check the validity of the derived model based on the assumed mode method.

One computer program available for the analysis of machine structure is COSMOS/M, finite element analysis software from Structural Research & Analysis Corp. COSMOS/M is a modular software package for design, analysis, and optimization, in which there are linear statics, dynamics, nonlinear, thermal, fluid, electromagnetic, and design

optimization modules. The module for linear dynamic analysis is used to evaluate the natural frequencies and corresponding mode shapes of the flexible manipulator system.

One example system was chosen for comparison, where the structural material is assumed to be same as that used in the flexible manipulator of the space shuttle.¹⁸ The lengths of the each link are assumed to be 7 m and the modulus of the elasticity is $14.5 \times 10^9 \text{ N/m}^2$. The density is assumed to be 1744 kg/m^3 . The outer diameter of the first link is 0.19 m and that of the second link is 0.15 m. The thickness is assumed to be 2 percent of the outer diameter of each link. The end effector or payload at the end of the link is assumed to be 100 Kgm and the mass of the second joint is 50 Kgm.

The two link manipulator is modeled by beam elements and mass elements in the FEM. The beam element is for a continuous mass model such as links and the mass element is for joint mass or end effector. All joints are assumed locked so that the rigid mode of beam is eliminated for the finite element analysis. The model in the FEM is clamped at the joints and the entire system is vibrating as a clamped free beam model. Each link is meshed into 10 elements of beam model. The program code for COSMOS/M appears in Appendix C.

The model in the assumed mode method uses only M_{ff} and K_{ff} to evaluate the flexible mode natural frequencies. The whole system is composed of two links and can be modeled as the summation of the two

clamped free beam models. Using only the flexible mode implies that the interactions between the rigid and flexible modes are not encountered as in the finite element analysis. Two vibration modes for each link means four vibration modes of the whole system in the assumed mode method and consequently, four flexible modes are calculated in the FEM for comparison. The results are shown in Table 3-1. The first two vibration modes from the AMM are about 4 percent different than the results from the FEM and the higher modes show bigger differences. The first vibration mode is the most important characteristic in the flexible mode so that only this mode of the beam will be used in the design stage. Figure 3-2 shows the first two mode shapes of the whole system from the finite element method.

Table 3-1 Frequency comparison of two methods and unit (Hz)

Mode	AMM	FEM	% difference
1	0.557	0.537	3.5
2	3.574	3.417	4.4
3	13.234	9.024	30.8
4	34.642	28.214	18.6

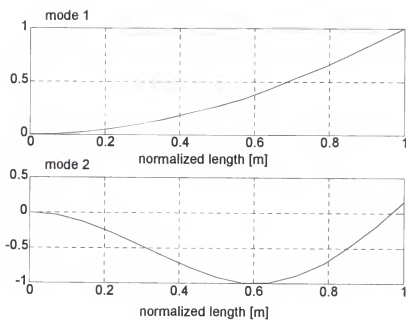


Figure 3-2 Mode shapes of the system from FEM
(Mode 1: 0.54 Hz and Mode 2: 3.4 Hz)

CHAPTER 4 STRUCTURAL OPTIMIZATION

Introduction

The optimization of a mechanical structure is an important step in the design process because there are many possible design results which satisfy the system's functional requirements within its design constraints. Some design variables may show contradicting effects which deteriorate one performance by increasing another. Optimization of a system is to find a structure that best satisfies the functional goal of the system within its design constraints.

For the manipulator system, there are two important characteristics which are the rigid and the flexible modes. The rigid mode characteristics include the dynamic parameters of the manipulator, such as velocity or acceleration, that depend on mass or inertia. The flexible mode characteristics are the deflection and vibration that affect the position accuracy and the speed limit of the manipulator. These two performance characteristics conflict with each other and the optimal point is a compromise between the two. Industrial robots focus on position accuracy by minimizing deflection and are, consequently, massive with poor rigid mode dynamics. Space manipulators have weight constraints and are,

generally, long and flexible with a large payload. Depending on the application of robots, the objective function should be appropriately chosen in the optimization scheme.

One of the common property used in objective functions is the eigenvalue, which is the natural frequency of the vibration and the pole in the control. This study includes this property for the optimal manipulator design. This chapter also considers optimization schemes based on the modal stiffness and impulse response. The natural frequency or modal stiffness should be chosen as high as possible to minimize deflection in the design of the machine structure but would result in a massive system with poor rigid mode dynamics. Therefore, a conflicting property such as weight minimization is included in the objective function to compensate for the adverse performance of a high frequency system.

This chapter is organized as follows. First, the structural characteristics of the manipulator are investigated. Second, mechanical optimization of the system is performed based on three performance characteristics: natural frequency, modal stiffness, and impulse response with the clamped free model. Clamped mass model is also optimized by using the natural frequency. The effects of the system parameters and the results of optimization schemes are discussed.

Structural Characteristics

The two link manipulator has two modes, the rigid and flexible body modes. Both modes have different effects so that this study investigates the system characteristics for both modes to find an optimal system. To improve the rigid mode performance, minimization of mass or inertia can be considered. In this study, minimization of mass will be used as the objective function to focus on light-weight characteristics of a space manipulator. For the flexible body mode, the structural characteristics of manipulators will be analyzed including the modal mass, modal stiffness and natural frequencies of the two link manipulator. Structural damping enhancement is one of the significant factors that can be used to reduce vibration but is beyond the scope of this study.

Mass

A system with large mass and inertia requires high-torque motors and consumes more energy to execute a motion and consequently a light-weight manipulator has benefits especially in space applications. There are popular light structural materials, such as magnesium, aluminum, and titanium, but the carbon composite material¹⁸ used in space manipulators is chosen for this study as described in the last section of Chapter 3. The external diameter of the link is limited in order to reduce waste of the usable space in the space shuttle. This study assumes that the external diameter is constrained to be within 30cm. The ratio of the thickness to

the external diameter is selected as 0.02 and it is assumed that no buckling occurs due to the negligible axial force. In reference, the space shuttle manipulator has less than 1 percent of this ratio but has stiffening rings inside of the links.

The objective is to reduce the mass and the objective function can be defined in normalized form as

$$U = \text{norm}(M) \quad (4-1)$$

where M is the mass of the two link manipulator and "norm" defines normalization of scalar values with the maximum value of its components. Figure 4-1 shows a contour plot of the objective function, Equation (4-1). As expected, the objective function values increase as the diameters of the links increase. The goal is to minimize the mass so that the minimum diameters for both links are the best selection for this objective function.

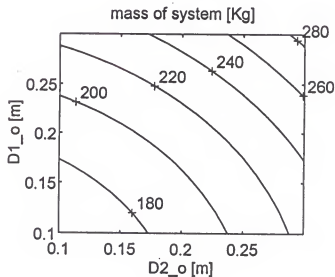


Figure 4-1 Contour plot of objective function for mass

Natural Frequency

Natural frequencies play important roles in the structural dynamics of a system. The dominant natural frequency of a mechanical structure should have the highest feasible value in order to reduce the transient response time and transient maximum vibration and to provide faster amplitude decay for a given damping characteristic. High natural frequency of a machine structure also yields high controller gain so that a wide bandwidth of controlled motion exists.

The natural frequencies can be calculated as

$$f_n = \frac{1}{2\pi} \sqrt{\text{eigenvalue of } [M]^{-1}[K]} \quad (4-2)$$

where unit of f_n is Hz and $[M]^{-1}[K]$ is the dynamic matrix. In calculation of the natural frequency, two cases can be considered based on conditions of joints. In the first case, the joints are locked with the whole system assumed to vibrate as a cantilever beam as in the FEM model in Chapter 3. The dynamic matrix uses only the flexible mode segments of the mass and stiffness matrices. In the second case, the joints are unlocked with the whole system as a combination of two cantilever beams. The dynamic matrix uses the rigid and flexible mode segments of the mass and stiffness matrices. The result will show four vibration modes in both cases and the joint-free case will show higher frequencies due to the pinned condition of the rigid mode at the joints.

The high number of vibration modes, generally, produces more accurate results for the simulation of the system characteristics. For the control application, the number of the vibration modes depends on the cut-off frequency of the controller. The exciting frequencies of external forces or disturbances also will influence the number of vibration modes to be considered in the design process. In this study, the first mode of vibration is considered to include the flexible mode in the design stage and four vibration modes are used in the time domain simulation of the response.

The dominant frequencies for both cases with the clamped free model are shown in Figure 4-2. The first mode of vibration shows very low frequencies (less than 1 Hz) for the joints-locked case. The dotted line represents the optimal line which has the highest frequency with minimum mass. The corresponding equation of this optimal structure is $D1=1.45*D2-0.006$ and the diameter of the first link is about 30 percent larger than that of the second link with joint-locked condition. Figure 4-2 (b) shows the natural frequencies with the joints-unlocked condition and the system shows much higher frequencies due to the pinned conditions at the first and second joints. The dotted line is the optimal line that provides the highest natural frequency with the lowest mass when the joints are free. The relation between the two diameters can be expressed as $D1= 1.14*D2-0.08$ which implies that the diameter of the second link should be larger than that of the first link. This is due to the payload at the end of the second link and to the clamped free conditions. The larger the payload the

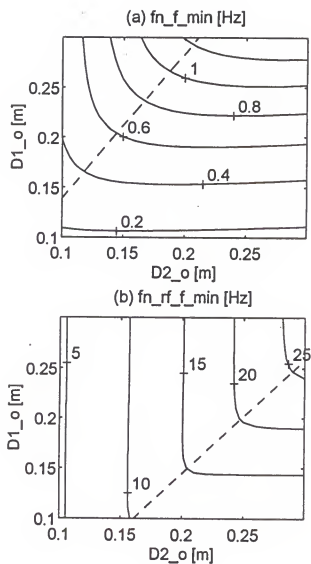


Figure 4-2 System natural frequencies with clamped free model
 (a) f_n with joints-locked; (b) f_n with joints-unlocked

larger the second link and a smaller payload will shift the optimal line upwards and, consequently, decrease the diameter of the second link. It can be noticed that increasing the diameters of both links increases the natural frequencies in both the joint-free and joint-locked conditions and also decreases the settling time of the response.

The mode shape then is changed to the clamped mass model and the natural frequencies are investigated. The results are plotted in Figure 4-3, and the natural frequencies in plot (a) for the joint-locked case show close values to those of the clamped free model in Figure 4-2 (a). When the joints are unlocked, the natural frequencies show much lower values when compared to the clamped free cases in Figure 4-2 (b). The two results show different values and the real value depends on the joint conditions. Figure 4-3 shows that the optimal line is $D1=1.45*D2-0.006$ for the joint-locked cases and $D1=1.26*D2-0.02$ for the joint-free cases.

Modal Stiffness and Modal Mass

Stiffness is one of the most important criteria for manipulator design due to its direct effect on position accuracy. Low stiffness results in large deflection of the link and longer settling times so that increasing stiffness is chosen as one of the objectives. The problem of stiffness enhancement is especially important where conventional techniques, such as increasing the size (which increases the mass) for higher stiffness, are unacceptable as in the case of space manipulators. The stiffness depends on material

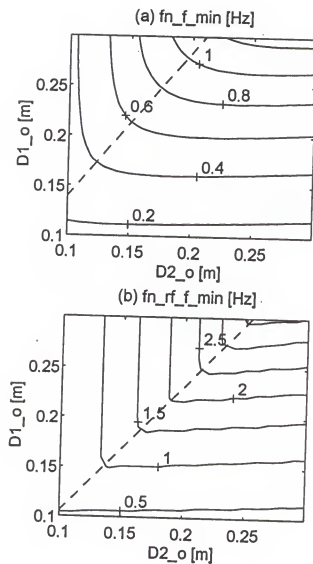


Figure 4-3 System natural frequencies with clamped mass model
(a) fn with joints-locked; (b) fn with joints-unlocked

property, length, and cross sectional area of the link so that one of them can be chosen to improve stiffness. The length depends on the work environment and the space manipulator usually has a long arm. The cross-sectional area is the only variable that affects stiffness in this study.

Modal stiffness can be calculated as follows:

- (a) Calculate modal matrix $[P]$ with $[M]$ and $[K]$
- (b) Rearrange the columns of $[P]$ in the order of ascending natural frequencies
- (c) Normalize modal matrix by the first element of each column of $[P]$
- (d) Calculate the modal stiffness matrix $[K_q] = [P]^T [K] [P]$

The modal stiffness of the structure is not constant and depends on the position of the input torque. Normalization by the first element of a column in (c) is valid when the input torque is on the first joint. When the input torque is on the second joint, normalization should be performed by using the second element of each column of the modal matrix. If the input is a disturbance at the end of the links, then normalization should be done by using the corresponding elements. It should be noticed that the physical meaning of the value of the modal stiffness depends on the units of that stiffness. The modal stiffness is calculated by using the modal matrix and the unit of modal coordinates are mixed with different units of generalized coordinates so that it is difficult to determine the exact physical meaning of the numerical value for modal stiffness.

Four modal stiffness values for the first vibration mode are calculated with the joint free model and plotted in Figures 4-4 and 4-5. Figure 4-4 shows the modal stiffness with two joint inputs, T1 and T2, so that these two cases are based on the joint-unlocked condition. The two inputs show different optimal lines, which are expressed as $D1=2.5934*D2-0.149$ for the T1 input and $D1=1.0839*D2-0.0643$ for the T2 input. The plots show expected results, namely, that the link with an input torque should be bigger than the other. Figure 4-5 shows the modal stiffness with disturbances at the end of the links with joint-locked condition. It should be noticed that these results are based on the disturbances for the first vibration mode of each link which is different than disturbances on the whole system. The resulting plot with these disturbances did not show any optimal lines but implies that the link with disturbances should have a larger diameter.

Modal mass with the clamped free model can be calculated in the same manner used in calculating the modal stiffness. The modal mass with torque inputs on the joints is calculated and the results are plotted in Figure 4-6. These results will be used to find the optimal mechanical structure in the next section.

Structural Optimization of Linkages

The selection of diameters for the optimum structure can be accomplished by solving the minimization problem for the selected

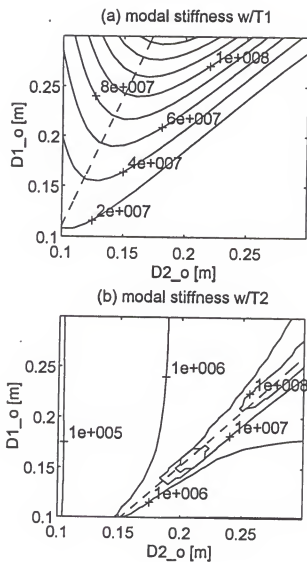


Figure 4-4 Modal stiffness with joint input

(a) Torque at the first joint

(b) Torque at the second joint

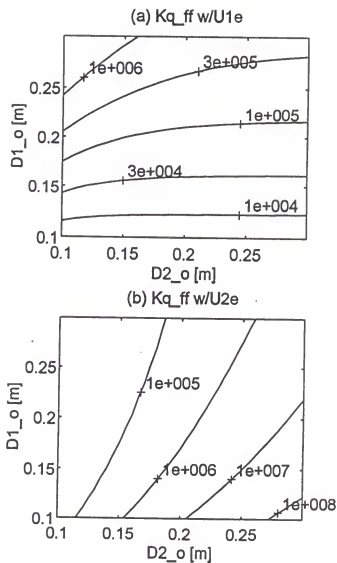


Figure 4-5 Modal stiffness with disturbance
 (a) Disturbance at the end of the first link
 (b) at the end of the second link

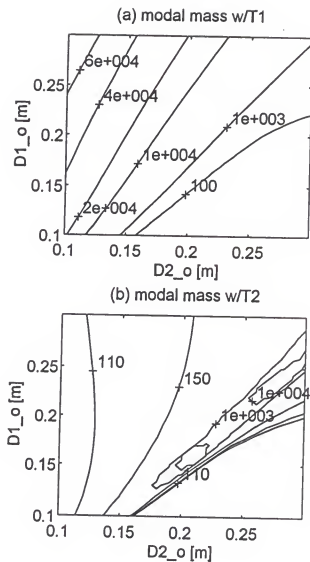


Figure 4-6 Modal mass with joint input

(a) Torque at the first joint

(b) Torque at the second joint

objective function. There are several schemes to find the solution of minimization problems. This study adopts the direct-search method so that the objective function is evaluated for all data points without using derivatives. The three objective functions used in this study are based on the natural frequency, impulse response, and modal stiffness.

Structural Optimization Based on Natural Frequency

The natural frequency is the most important property for an oscillatory system. This structural characteristic affects settling time, position accuracy, and the cut-off frequency of the controller. The objective function considered in this section includes the natural frequency for better response and the mass to minimize weight of the system. Two different natural frequencies are considered that correspond to the joint-locked and joint-free cases as shown in Figure 4-2. These two frequencies are combined to develop new contour plots and an optimal structure can be found by adding mass. The objective function, therefore, is written as

$$U_1 = \text{norm}(M) + SF_1 \times \left(\text{norm}\left(\frac{1}{f_{n\_f}}\right) + SF_2 * \text{norm}\left(\frac{1}{f_{n\_rf}}\right) \right) \quad (4-3)$$

where $SF_{1,2}$ are scale factors.

The significance of the different objectives can be scaled by the weighting factor or scale factor. This study uses two assumptions: first, the two natural frequencies have equal importance and, second, the mass

and natural frequency also have the same significance. The general way of scaling due to the difference of variation can be written as

$$SF = \frac{(\max - \min) \text{ of variable 1}}{(\max - \min) \text{ of variable 2}} \quad (4-4)$$

This scale factor can become unreasonable when components of the variables have extreme values due to nonlinear properties. These extreme values should be excluded in the calculation of the scale factor and, in some cases, the mean value of two variables can be used in the scaling. No systematic way exists to find the scale factor, but Equation (4-4) is chosen to give a reasonable value in this study.

Two contour plots of the performance index are shown in Figure 4-7. The static case with the joints-locked as illustrated in Figure 4-2 showed that the diameter of the first link should be larger than that of the second link and the dynamic case with joints-free showed opposite results. When two natural frequencies are combined with equal importance in the objective function, the scale factor, SF_2 , was found to be 0.95. The resulting optimal line in Figure 4-7 (a) is $D1 = 1.0539 * D2 - 0.0107$ and suggests that the two diameters of the manipulator should be almost the same. Gravity is not considered here and the effect of gravity may shift the optimal line upwards to decrease inertia by reducing the size of the second link. When the diameter of the first link is less than 0.15m, values of the objective function change sharply and this design region is undesirable due to the high sensitivity to change in the design variables. This plot shows

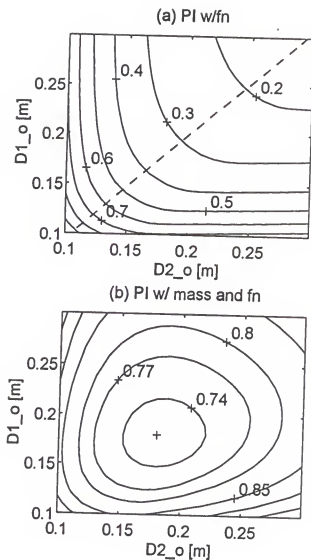


Figure 4-7 Contour plot of PI with clamped free model

(a) PI with frequencies (joint locked and unlocked)

(b) PI with mass and frequencies

robust design with small variation of the objective function when the diameters are larger than 0.18m. The goal is minimization of the objective function and larger diameters of both links show better performances without mass consideration in Figure 4-7 (a). The best structure in this case can be selected with diameters, $D1=0.3\text{m}$ and $D2=0.3\text{m}$, where these are constrained by their upper limits.

When mass minimization is included in the objective function, large diameters for both links reduce the design merit. The scale factor SF_1 is calculated based on Equation (4-4). The minimum and the maximum values of two variables are chosen along the optimal line and SF_1 was calculated to be 0.585. Optimal diameters of the two link manipulators based on this weighting factor were found to be $D1= 0.18\text{m}$ and $D2=0.18\text{m}$ as in the contour plots in Figure 4-7 (b). The numbers on the curves represent the calculated objective function based on Equation (4-3) and this value at the optimal point is 0.729. The mass of the optimum two links in Figure 4-7 (b) is decreased by 64 percent from the optimal case in Figure 4-7 (a). This final optimum design has the same size of link diameters and is well within the constraints imposed on the diameters. The corresponding system natural frequencies are 0.538 Hz for the joint-locked case and 12.71 Hz for the joint-free case.

The same optimization scheme was performed with the clamped mass model for comparison and the results are plotted in Figure 4-8. When two natural frequencies are combined, the line equation of the optimal structure

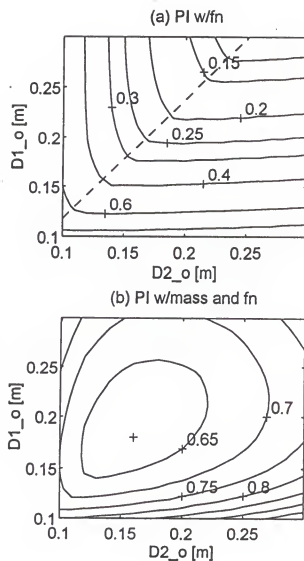


Figure 4-8 Contour plot of PI with clamped mass model
 (a) PI with frequencies (joint locked and unlocked)
 (b) PI with mass and frequencies

based on frequency consideration is $D1=1.2059*D2-0.003$ with scale factor SF_2 0.99 in Figure 4-8 (a). This result implies that the optimal first link diameter is 16 percent larger than that of the second link. After the mass is added with scale factor SF_1 0.59, Figure 4-8 (b) shows the optimal solution to be $D1=0.18\text{m}$ and $D2=0.16\text{m}$. The scale factor is calculated in the same way as in the previous case. The natural frequencies of this structure are 0.485 Hz for the joints-locked case and 1.425 Hz for the joints-free case. This structure has same size for the first link and 11 percent smaller diameter for the second link compared to the optimal structure for the clamped free model in Figure 4-7 (b). Optimization of the clamped mass model reduced system inertia by decreasing the size of the second link. The high gradients in Figure 4-8 (b) imply that smaller diameters of the first link are inadvisable for the optimal structure due to the sensitivity of the design to diametric variations.

Structural Optimization Based on Modal Stiffness

Stiffness is an important characteristic due to its direct effects on manipulator accuracy. This section considers an optimization based on modal stiffness with mass effect. The stiffness depends on general input location and will be investigated with two torque inputs and two disturbances. The optimization scheme uses the dynamic stiffness of the first mode of vibration. Static stiffness is also an important characteristic but it is excluded here due to neglecting gravity for the space manipulator.

The objective function includes the modal stiffness; because the goal is minimization of the objective function, the inverse of the modal stiffness is used. System stiffness depends on the location of the general torque input. Since there are two torque inputs, the objective function uses a combination of these inputs. By adding the mass effect, the objective function is written as

$$U_2 = \text{norm}(M) + SF_1 \times \left(\text{norm}\left(\frac{1}{kq}\right) \Big|_{w/T1} + SF_2 * \text{norm}\left(\frac{1}{kq}\right) \Big|_{w/T2} \right). \quad (4-5)$$

This study assumes that the two torque inputs, the mass and the stiffness have equal importance. Accordingly, the scale factors are based on the range of the variation of the mass and the two modal stiffness.

First, the two modal stiffness results in Figure 4-4 are combined; the resulting merit contours are plotted without the mass effect in Figure 4-9 (a). The range of the variation of the variables was chosen from the optimal lines in Figure 4-4; the scale factor SF_2 used in this case is 0.049. The optimal line for the stiffness is $D1=1.539*D2-0.095$ with the general torque inputs. The high gradients at the lower right side indicate that a smaller-diameter first link and a larger-diameter second link are poor design selections. The best structure without consideration of mass is $D1=0.3\text{m}$ and $D2=0.257$. The contour plots in Figure 4-9 (b) show that the optimal diameters are $D1=0.18\text{m}$ and $D2=0.18\text{m}$ when including the effect of the mass with scale factor SF_1 16.8. This factor was calculated as in

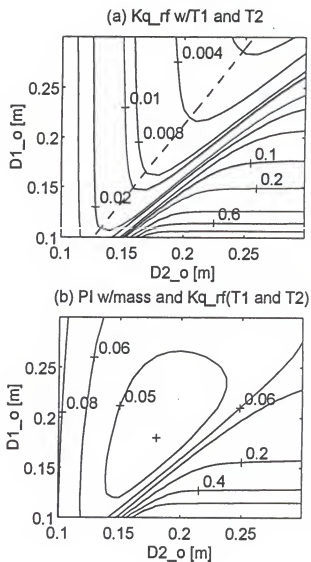


Figure 4-9 Contour plot of PI with torque inputs
 (a) PI with stiffness; (b) PI with mass and stiffness

the previous cases. This optimal structure shows the same size for the two links and its calculated objective function is 0.046.

Stiffness also differs when there are general disturbances at the end of the joints. This study assumes that the effects of these general disturbances are identical. The same objective function is used as in Equation (4-5), but the system inputs are changed from the general joint torque inputs to the general disturbances at the end of each link. The two plots in Figure 4-5 are combined with a scale factor SF_2 1.00 based on Equation (4-4). The resulting iso-merit contours for the stiffness optimization are plotted in Figure 4-10 (a). The optimal line based on stiffness is $D1=1.169*D2-0.001$ for the general disturbances. The contour plots in the figure imply that the first link is 14 percent larger than the second link based on the disturbance effects. The contour plots in Figure 4-10 (b) include the effect of the mass with scale factor SF_1 1.88. This factor is calculated as in the previous cases. The optimal structure is found to have $D1=0.17m$ and $D2=0.15m$ with a difference between the two diameters of approximately 12 percent. The calculated objective function is 0.539 at this optimal point.

Structural Optimization Based on Impulse Response

The optimization for the minimization of the vibration amplitude can be pursued by using the impulse response. The response of a second order system excited by an impulse $\hat{I}$ can be written as

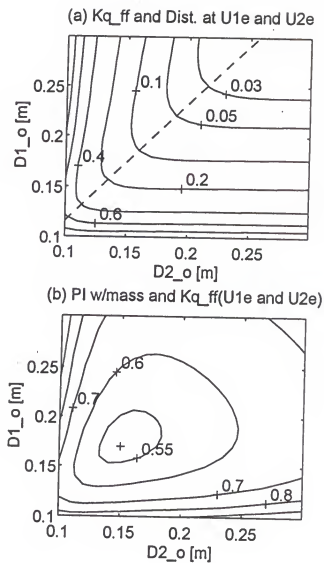


Figure 4-10 Contour plot of PI with disturbances
 (a) PI with stiffness; (b) PI with mass and stiffness

$$x(t) = \frac{\hat{I}}{m \times \omega_n} \sin(\omega_n t) = \frac{\hat{I}}{\sqrt{m \times k}} \sin(\omega_n t). \quad (4-6)$$

This equation implies that higher values of the product of modal mass and modal stiffness decrease the vibration of a structure. This also means that one of these properties -natural frequency, stiffness, or mass- can be chosen to decrease the oscillation. If damping is considered, then the response contains the exponential term representing the decay of the transient response. Note that the amplitude of the steady state response of a second order system at the natural frequency is

$$X = \frac{F}{2k\zeta} \quad (4-7)$$

where X and F are the magnitude of response and input force and ζ is the damping ratio. This response shows that the magnitude is only a function of the stiffness and the damping. The modal mass and the natural frequency do not effect the magnitude of the vibration at resonance. Accordingly, if the system has external steady inputs such as in a machining environment, then Equation (4-7) is important. This study assumes that there are no steady inputs and that transient vibration response is the only important consideration in flexible manipulator dynamics. Therefore, minimization of the impulse response is chosen for the objective function.

Two different inputs, i.e., any general torque inputs and any general disturbances, are investigated. The modal parameters with two joint inputs are used from Figure 4-4 and 4-6. For input at the first joint, the modal

products are plotted in Figure 4-11 (a). The optimal line in this case is $D1=4.693*D2-0.303$ for the minimum vibration with minimum mass. The best structure on this line has $D1=0.3m$ and $D2=0.13m$ where $D1$ is constrained by its upper limit and the difference of two diameters is 57 percent. This structure shows the largest difference in size between the two diameters. When the input is at the second joint, the optimal line for the minimum weight and vibration is $D1=1.101*D2-0.067$. This implies that a larger diameter of the second link is preferred for inputs at the second joint. Figure 4-12 shows the contour plot with disturbances at the end of each link based on first mode response. There are no optimal lines in the contour plot in this case. Since the disturbances are affecting the first mode of each link, the response improves by increasing the diameter of the link that is subjected to the disturbance.

Since higher values for the product of modal mass and modal stiffness produce a smaller vibration response, the inverse of this product is used for the objective function in the minimization problem. This study assumes that two products plotted in Figure 4-11 and 4-12 have equal importance when combined in the objective function. First, the modal parameters are combined from two plots of Figure 4-11 and 4-12 with scale factor SF_2 . After including the mass of the manipulator with scale factor SF_1 , the objective function can be written as

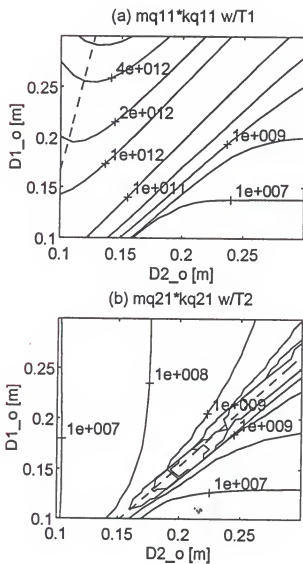


Figure 4-11 Product of modal mass and modal stiffness with torque inputs
 (a) $m_q \cdot k_q$ with T1; (b) $m_q \cdot k_q$ with T2

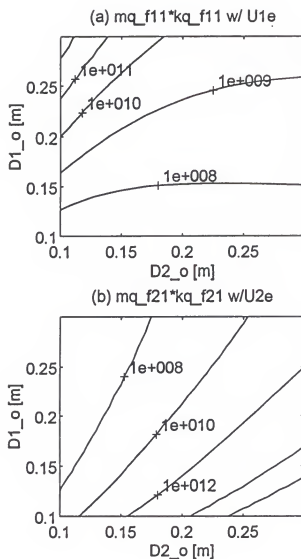


Figure 4-12 Product of modal mass and modal stiffness with disturbances

(a) $m q * k q$ with disturbance at $U1e$

(b) $m q * k q$ with disturbance at $U2e$

$$U_3 = \text{norm}(M) + SF_1 \times \left(\text{norm}\left(\frac{1}{mq \times kq}\right) \Big|_{w/T1} + SF_2 * \text{norm}\left(\frac{1}{mq \times kq}\right) \Big|_{w/T2} \right) \quad (4-8)$$

The system subjected to general torque inputs is first investigated. The scale factor is calculated based on Equation (4-4) and the range is chosen from the optimal lines. The scale factor SF_2 is $1.638e-4$ in Equation (4-8) and the resulting contours without mass are plotted in Figure 4-13 (a). The optimal line in this case can be determined as $D1 = 1.959 * D2 - 0.1275$ and the optimal point is $D1 = 0.3m$ and $D2 = 0.22m$ where $D1$ is constrained by its upper limit. After including mass in the objective function, Figure 4-13 (b) is plotted based on Equation (4-8) with scale factor $SF_1 = 1.146e4$ and the optimal structure has $D1 = 0.18m$ and $D2 = 0.16m$ with the calculated objective function as $6.678e-5$. There is a 68 percent decrease in the mass of the optimal structure when the mass is included in the objective function.

The effect of the disturbances with the joint-locked is investigated here by changing the system inputs from general joint torque to general disturbances. The two plots for the modal parameters in Figure 4-12 are combined with scale factor $SF_2 = 1.00$. Figure 4-14 (a) shows that the optimal line is $D1 = 1.403 * D2 - 0.0044$ for the minimal vibration for disturbances at the end of each link. After including mass effects with scale factor $SF_1 = 2.19$, the contours are plotted in Figure 4-14 (b). It can

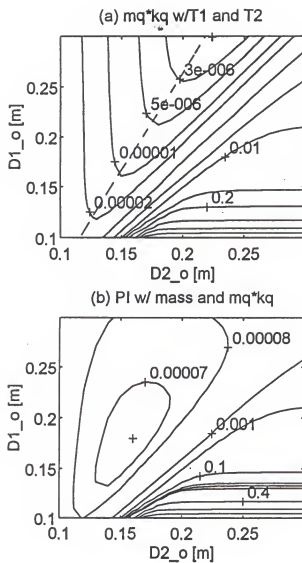


Figure 4-13 PI based on the product of modal mass and stiffness with general joint torque inputs

(a) PI with $m\dot{q}^*k\dot{q}$ with T1 and T2; (b) PI with mass and $m\dot{q}^*k\dot{q}$

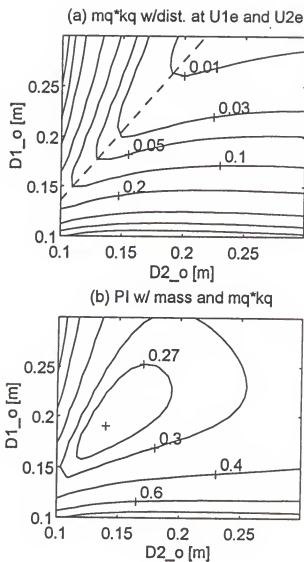


Figure 4-14 PI based on the product of modal mass and stiffness with general disturbances

- (a) PI with disturbances at ends of the each link
 (b) PI with mass and disturbances

be seen that the optimal structure has $D1=0.19\text{m}$ and $D2=0.14\text{m}$ and that the calculated objective function is 0.253 at the optimal point.

Summary and Discussion

A structural optimization procedure was performed in this chapter based on the natural frequency, modal stiffness, and modal mass for general torque and general disturbance inputs. The study performed here is different from designing a general industrial manipulator because of several assumptions:

- (a) There is no gravity in the planar motion so that static stiffness is excluded. This would be inadequate if a long-arm manipulator were used in the gravitational field.
- (b) Two natural frequencies only are considered and are assumed to have the same importance whether the joints are locked or free.
- (c) Mass and the other system dynamic properties (f_n , k_q , $m_q \cdot k_q$) are assumed to have the same importance in selecting link diameters. The weighting factors of these different properties in general would depend on the application of the robot.
- (d) The responses of the system to the two general torque inputs or the two general disturbances inputs are assumed to be equally important.

These assumptions as well as the choice of the scale factors clearly produce different results.

Another important consideration is the beam models, e.g., clamped free and clamped mass. In reality, the modeling would depend on the joint conditions; accordingly, this study adopted the two models for frequency optimization. The dominant natural frequencies of the two models were almost same for the joint-locked condition but were quite different for the joint-free condition. The optimal relationship between the link diameters was found to be linear and were plotted for the highest frequencies with minimum mass in Figure 4-15. The two models show different optimal linear relationships and the clamped mass model has larger first link and smaller second link diameters compared to the clamped free model. This implies that more independent and complex joint conditions would increase the ratio between the diameters. When there is a mass constraint of 200 Kgm on the whole system, the clamped free model has same-sized links

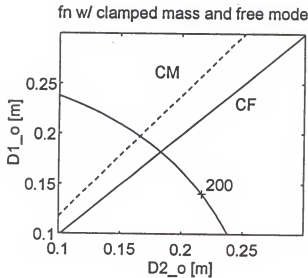


Figure 4-15 Optimal lines with different mode shapes

while the clamped mass model shows 16 percent difference between the two diameters.

Two optimization objectives, namely, the natural frequency and stiffness, are commonly used in structural design. The impulse response is normally used in control and vibration theory. Both of these objectives are considered for the structural optimization in this study. Optimization based on modal stiffness and impulse response showed that the optimal solution depends on input torque location. The link with a torque input or a disturbance input is found to be larger than the other. Two performance indexes were combined with a scale factor and the results are plotted for comparison in Figure 4-16 based on the clamped free model. The plots indicate that including the modal mass requires that the system have a larger-diameter first link and a smaller-diameter second link in both cases. Figure 4-17 shows the optimal lines for the joint-locked case with the clamped free model based on the flexible mode mass and stiffness matrices. The plot in Figure 4-17 (a) shows close results for the two optimal lines based on the dominant flexible mode frequency and on the product of the modal mass and the modal stiffness with disturbances at the end of each link. The plot in Figure 4-17 (b) shows different results for the optimal lines based on the dominant flexible mode frequency and on the modal stiffness with disturbances at the end of the each link. The optimization based on the modal stiffness produces a larger diametric ratio $D2/D1$ for the links.

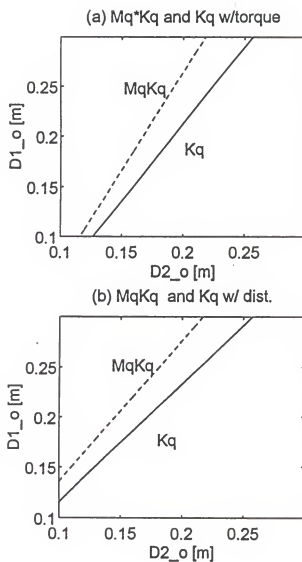


Figure 4-16 Optimal lines with different objective functions

(a) Optimal lines with torque inputs

(b) Optimal lines with disturbances

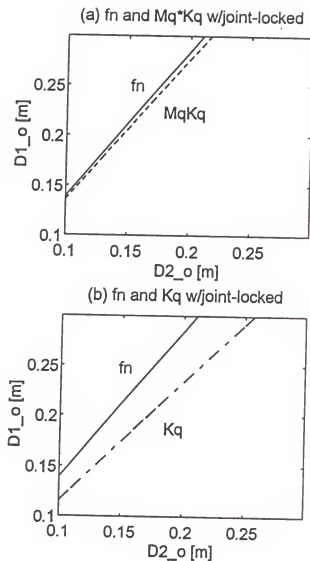


Figure 4-17 Optimal lines with different objective functions with joint-locked condition

- (a) Optimal lines based on frequency and product of the modal mass and modal stiffness
- (b) Optimal lines based on frequency and stiffness

CHAPTER 5 OPTIMIZATION OF SYSTEM WITH CONTROLLER FOR ROBUST DESIGN

Introduction

A methodology for the design of a two link manipulator by using a controller in the design stage is proposed in this chapter. In general, the structural design and the controller design are not commonly coupled. The structural designer performs the manipulator design using structural characteristics. The goal of the structural designer is to find an optimal system that satisfies the mechanical constraints. The previous chapter is an example of structural optimization from the mechanical viewpoint. The controller designer is generally uninvolved in the mechanical design process. He incorporates a mechanical structure in the control loop and finds a controller that maximizes manipulator performance. Although the mechanical structure and controller are optimized independently in this uncoupled design approach, the results do not guarantee that the mechanical structure is optimal within the controller loop.

The optimal structure as determined in the previous chapter is based on the objective functions of using the mass and the flexible mode characteristics so that it basically utilizes only mechanical criteria. The proposed design approach proceeds to modify the dimensions of the

structurally optimized system by including the controller effect. If the controller selection is based on the rigid mode, then including the controller in the design stage has the equivalent effect of including the rigid mode in the structural design. If it is based on the flexible mode, then the controller effects will improve the flexible mode. This study uses a PD (proportional and derivative) controller based on the rigid mode. The rigid mode performance is governed by the controller and the flexible mode performance is governed by the structural characteristics.

The PD controller provides structural benefits when utilized in the design stage. The proportional gain behaves as a stiffness and the derivative as a damper in the rigid mode. Therefore, including the PD controller completes the model of a manipulator system by allowing it to have mass, damping, and stiffness in the rigid mode. The modal approach can be used to derive the structural parameters for the rigid mode even though these are based on the behavior of the electric circuit of the controller. The resulting manipulator has all structural parameters in both the rigid and flexible modes.

As mentioned in the literature review, Park and Adada⁴¹ included the controller in the design of a two link manipulator by using the rigid mode frequency. This study proposes a different approach by including the controller effects in the structural optimization process. The structural optimization discussed in the previous chapter will be extended to consider the rigid mode.

This chapter is organized in the following order. A simple controller will be constructed based on the classic control theory and the system will be formulated to include the controller effects. The optimal structure, based on the natural frequencies from the previous chapter, will be used as the initial structure. Both the clamped free and clamped mass models are used to check the effect of the developed concept on each model. After determining new optimal mechanical structures, their performances will be compared with that of the initial structures for the rigid and flexible modes.

Controller Design

This section presents the controller design procedure. The PD controller used in this study has two gains for each link so that four gains should be determined according to appropriate controller design methods. One of the general methods is pole assignments and the control gains can be chosen by assigning poles that are functions of the natural frequency and damping. The controller frequency of the rigid mode, generally, is constrained to be less than a half of the lowest natural frequency of the flexible mode in order not to excite the dominant flexible mode.¹⁰ The optimal damping ratio for the second order system is 0.707 based on the ITAE (the integral of time multiplied by the squared error) criterion. This damping ratio also guarantees that the magnitude of the nondimensional transfer function between spring force due to dynamic displacement and the input force is less than 1 for any steady state exciting frequency. These

conditions are used to calculate the controller gains. In reality, the differential controller is not used alone without additional low-pass filters because of stability problems. It is used here as a tool in the design stage and not for actual control of the manipulator.

Three considerations are assumed in the simulation of the control system. First, there is an upper limit for the input torque on the manipulator that has the equivalent effect of voltage or current limit. Second, the electric time constant of the motor is assumed to be much faster than the mechanical time constant. This makes it possible to treat the motor itself as a constant gain instead of a first order system so that the motor can be modeled as a part of the proportional controller gain. Third, the position and velocity control operate in the continuous time domain so that high frequency effects are detectable in the simulation due to the very small time steps. The time step normally depends on the time constant of the system so that the sampling frequency is chosen at least 10 times higher than the highest natural frequency of the whole system.

The initial system used is the structurally optimized manipulator as determined in last chapter with the clamped free model. The diameters are 0.18m for both links and the dominant natural frequency is 0.5369 Hz for the joint locked case and 12.6256 Hz for the joint free case. The natural frequency for the joint-locked case is used to calculate the cut-off frequency of the controller. The controller gains are decided based on this

initial system and the resulting controller gains will be used to include the controller effects on the system design.

The PD controller has two gains for each joint so that four gains should be determined for the whole system. Figure 5-1 shows the block diagram of the whole system with the PD controller and the flexible manipulator. The control loop contains two structural and four control design variables. The controller can be selected based on either rigid mode or flexible mode characteristics, but only the rigid mode will be used for the controller design in this study. The resulting controller has two natural frequencies for the rigid mode, which are relatively low compared to the flexible mode frequency. They will not excite the dominant flexible mode.

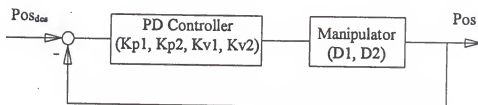


Figure 5-1 Control system

The control gains are designed according to the following procedure. The rigid mode mass matrix (M_{rr}) and independent joint controller (two proportional and two derivative gains) can be written as

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{Bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{Bmatrix} + \begin{bmatrix} K_v1 & 0 \\ 0 & K_v2 \end{bmatrix} \begin{Bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{Bmatrix} + \begin{bmatrix} K_p1 & 0 \\ 0 & K_p2 \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix} = 0 \quad (5-1)$$

where K_{p1} and K_{p2} are the proportional gains for the first and the second links and K_{v1} and K_{v2} are the derivative gains for the two links. This system is coupled dynamically because of the mass matrix, which can be converted into two independent modal equations. The pole assignment can be used to make the system exhibit the desired structural properties. The rigid mode controller has two natural frequencies. The higher natural frequency of the controller is selected to be close to one half of the lowest natural frequency of the flexible mode with joint locked; the lower natural frequency of the controller is chosen to be as high as possible. These two controller design objective can be used to find two proportional gains, K_{p1} and K_{p2} . The resulting objective function can be written as

$$U_4 = \text{norm}\left(\frac{1}{f_{n\_r1}}\right) + W_{F1} \times \text{norm}\left(1 - \frac{f_{n\_r2}}{f_{n\_r2\_d}}\right) \quad (5-2)$$

where "norm" is normalization of a matrix with respect to its maximum value; f_{n_r1} and f_{n_r2} are the first and the second mode frequencies of the controller; $f_{n\_r2\_d}$ is the desired second mode frequency of the controller, which is half of the first flexible mode frequency; and W_{F1} is the weighting factor.

The derivative gains of the controller will change the damping of the system. One consideration in finding the derivative gains is that the damping ratio of the first mode of the controller is chosen to be close to 0.707. Another consideration is to minimize the coupling of the two rigid

modes in the modal space by decreasing the off-diagonal terms of the modal damping matrix. Accordingly, the objective function for these considerations can be staged as

$$U_5 = \text{norm}\left(1 - \frac{\zeta_1}{0.707}\right) + \text{WF}_2 \times \text{norm}(\text{[off - diag of [Cq]])} \quad (5-3)$$

where ζ_1 is the damping ratio of the first mode of the controller and [Cq] is the modal damping matrix.

The normalized objective functions based on Equation (5-2) and (5-3) are calculated with a range of controller gains. The weighting factors WF_1 and WF_2 are chosen as 1 for both Equations (5-2) and (5-3) and the resulting contours of the performance index are plotted in Figure 5-2. The optimal proportional controller gains are found to be $\text{Kp1}=4700$ and $\text{Kp2}=1100$, which correspond to controller frequencies of 0.0505 Hz and 0.2686 Hz in the modal space. The derivative gains are $\text{Kv1}=21000$ and $\text{Kv2}=4900$ and the corresponding damping ratio is 0.7074 for the first mode of the controller. Equations (5-4) and (5-5) show the resulting formulation in the general coordinates and the modal coordinates respectively.

$$\begin{bmatrix} 25333 & 10895 \\ 10895 & 5398 \end{bmatrix} \begin{Bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{Bmatrix} + \begin{bmatrix} 21000 & 0 \\ 0 & 4900 \end{bmatrix} \begin{Bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{Bmatrix} + \begin{bmatrix} 4700 & 0 \\ 0 & 1100 \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix} = 0 \quad (5-4)$$

$$\begin{bmatrix} 89015 & 0 \\ 0 & 3474 \end{bmatrix} \begin{Bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{Bmatrix} + \begin{bmatrix} 39931 & 64 \\ 64 & 44155 \end{bmatrix} \begin{Bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{Bmatrix} + \begin{bmatrix} 8950 & 0 \\ 0 & 9898 \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = 0 \quad (5-5)$$

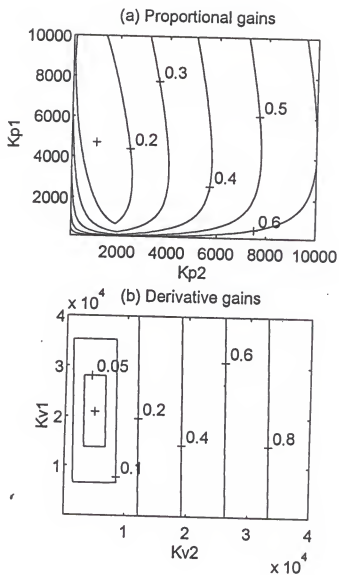


Figure 5-2 Contour plot for control gains
 (a) PI for proportional gains
 (b) PI for derivative gains

The magnitude of the off-diagonal term of the modal damping matrix is 64, which is 0.16 percent of the smaller diagonal term (39931) of the modal damping matrix. The resulting damping ratio is 3.76 for the second mode of the controller. The programs used for the controller design are listed in Appendix D.

Structural Optimization Including Controller Effects

This section presents the procedure of including the controller effects in the system design with the clamped free model. The controller used in the process is based on the rigid mode so that the system performance of the rigid mode is expected to be improved. The three optimization schemes used in the structural design of the last chapter are extended to include the controller effects. Accordingly, the performance indexes are expressed as functions of the natural frequency, of the modal stiffness, and of the product of the modal mass and modal stiffness in the rigid mode control system.

When performance-index variables are considered in the rigid mode, they should be changed to reflect the different characteristics of the rigid and the flexible modes. For example, a higher value of the product of the modal mass and modal stiffness would be desirable in order to minimize the vibration in the flexible mode. On the other hand, a lower value is preferred for improving the system response in the rigid mode. The modal stiffness in the rigid mode should be interpreted differently than in the

flexible mode. The higher modal stiffness in the flexible mode implies a stiffer link or smaller structural vibration. The higher stiffness in the rigid mode implies a powerful and stiff controller with higher proportional gains.

The Effect of Including Natural Frequency of Controller

First, the natural frequency of the rigid mode controller is used to illustrate the contributions that the controller can make to the structural optimization process. The two proportional gains are fixed and the rigid mode natural frequencies are calculated by changing the diameters within the constraints. The first mode frequency of the controller is plotted in Figure 5-3 (a). The frequency of the rigid mode is much more dependent on the diameter of the second link and the first link has little effect on the frequency. The frequencies in Figure 5-3 (a) do not show significant change because the main factor of this frequency is the lengths rather than the diameters of the manipulator. If the effect of this natural frequency is included, the diameter of the second link is expected to decrease more than the first link.

The inverse of this natural frequency is used for the minimization problem and its normalized values are incorporated in the objective function. The resulting objective function is the summation of the first rigid mode frequency effects and the flexible mode effects from the objective function U_1 in Equation (4-3). It can be written in the following form

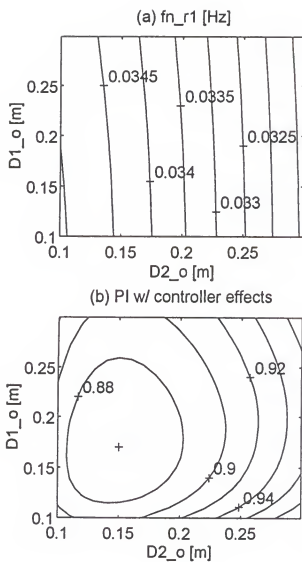


Figure 5-3 Including the natural frequency of the controller
 (a) First mode frequency of the rigid mode control system
 (b) PI with controller effects

$$U_6 = \text{norm}\left(\frac{1}{f_{n\_r1}}\right) + SF_3 \times \text{norm}(U_1) \quad (5-6)$$

where SF_3 is the scale factor based on Equation (4-4) and f_{n_r1} is the first mode controller frequency from Figure 5-3 (a).

The normalized values of the performance index U_6 are plotted in Figure 5-3 (b) with the calculated scale factor SF_3 0.40. The new optimal structure is found to have $D1=0.17\text{m}$ and $D2=0.15\text{m}$ with a merit value of 0.8649. This new optimization scheme decreased the diameters by 5.5 and 16.7 percent for the first and second links respectively. The new optimal structure obtained with the inclusion of the controller effects shows greater change in the diameter of the second link than that of the first link. Including the natural frequency of the controller should guarantee that this new structure has better performance in the rigid mode, although improvement in flexible mode performance is uncertain. The performances of the initial and new systems will be compared in the next section.

The Effect of Including Modal Stiffness of Controller

The modal stiffness of the rigid mode controller is considered here to illustrate another way of including the controller effects. The proportional gains of the controller remain fixed while the diameters of the links are changeable within the selected constraints. The modal stiffness of the first rigid mode is calculated by changing the diameters with the two fixed proportional gains. The results are presented in Figure 5-4 (a) with iso-

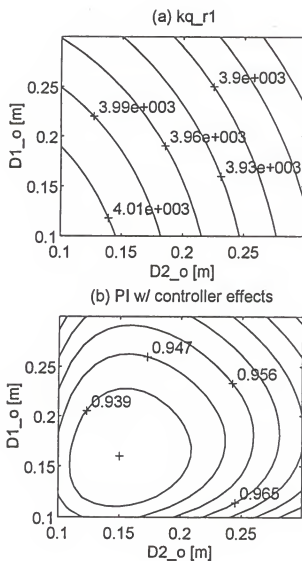


Figure 5-4 Including the modal stiffness of the controller
 (a) First mode modal stiffness of the control system
 (b) PI with controller effects

merit contours. The higher value of the modal stiffness can be explained by proportional gains having more powerful effect on a given structure. The best direction for an initial structure to have a stiff and powerful controller is the normal line to the iso-merit contours.

The higher stiffness always is preferred so that the inverse of the modal stiffness of first mode is used for the minimization problem and its normalized value is incorporated in the objective function. After including the objective function U_1 in Equation (4-3), the resulting objective function can be written in following form

$$U_7 = \text{norm}\left(\frac{1}{kq_{r1}}\right) + SF_3 \times \text{norm}(U_1) \quad (5-7)$$

where kq_{r1} is the modal stiffness of the first mode of the controller and

$$U_1 = \text{norm}(M) + SF_1 \times \left(\text{norm}\left(\frac{1}{fn_f}\right) + SF_2 * \text{norm}\left(\frac{1}{fn_{rf}}\right) \right).$$

It is expected from Figure 5-4 (a) that including the modal stiffness of the controller will decrease the diameter of both links. The scale factor SF_3 is calculated based on Equation (4-4) and found to be 0.214. By using Equation (5-7), the objective function U_7 is calculated and plotted in Figure 5-4 (b). The new optimal diameters were found to be $D1=0.16\text{m}$ and $D2=0.15\text{m}$ with a merit value of 0.9297. The optimal diameters are decreased by 11.1 and 16.7 percent for the first and second links respectively.

The Effect of Including Product of Modal Mass and Modal Stiffness

The impulse response can be used to include the effects of the rigid mode controller. The impulse response can represent the total effect for both the rigid and flexible modes. Theoretically, the impulse input rarely is applied directly to the manipulator system, but the impulse response generally is useful in finding an optimal structure with minimal vibration. In other words, the product of modal mass and modal stiffness is one of the variables that can improve the performances of both the rigid and flexible modes. The scalar value of this product has different effects in both modes; a higher value is preferred to minimize vibration of the flexible mode, but a lower value is preferred for the fast rigid mode system.

The product of the modal mass and modal stiffness is calculated by using the first mode of the controller and is plotted in Figure 5-5 (a). The goal is to minimize the value of this product for better rigid mode performance. It can be seen from Figure 5-5 (a) that including this product will increase the diameter of the first link but decrease that of the second link. The objective function to include the controller effects will use the normalized value of this product and can be written in following form

$$U_8 = \text{norm}(mq\_r1 \times kq\_r1) + SF_3 \times \text{norm}(U_1) \quad (5-8)$$

where mq_r1 is the first mode modal mass of the control system.

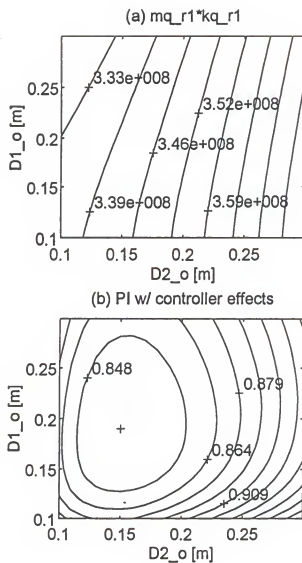


Figure 5-5 Including the modal product of the controller
 (a) Product of the modal mass and the modal stiffness
 (b) PI with controller effects

The performance index is calculated based on Equation (5-8) and the scale factor SF_3 used is 0.5671 based on Equation (4-4). The normalized value of this objective function is plotted in Figure 5-5 (b). The new optimal structure has $D1=0.19\text{m}$ and $D2=0.15\text{m}$ with a merit value 0.8333. This procedure increased the diameter of the first link by 5.5 percent and decreased that of the second link by 16.7 percent.

Comparison of the Results

This section gives comparisons of the obtained results from several viewpoints including the time domain simulation. The clamped free mode is used in the last section and the results from the clamped mass model also is discussed in this section.

All three schemes in the last section considered controller effects based on the rigid mode. Therefore, it is expected that the rigid mode characteristics of the system likely will be improved due to the controller effects. The resulting structures indicate that all three schemes produced a smaller diameter for the second link, while the diameter of the first link varied depending on the optimizing scheme. The scheme based on natural frequency and modal stiffness produced a smaller diameter for the first link. The third scheme employing the impulse response indicated that a larger first link and a smaller second link will improve the rigid mode dynamics of the manipulator.

The total response of the system, including the rigid and flexible modes, to a step input also is investigated. The overshoot for the rigid mode and the deflection of the end-effector, u_{end} , for the flexible mode are calculated based on the following equations:

$$\text{OS (\%)} = \left| \frac{\text{maximum distance} - \text{desired distance}}{\text{desired distance}} \right| \times 100 \quad (5-9)$$

$$u_{\text{end}} = u_{1E} \times \cos(\theta_2) + u'_{1E} \times L_2 + u_{2E} \quad (5-10)$$

where the distance in Equation (5-9) is in x-y coordinates and $\cos(\theta_2)$ in Equation (5-10) gives the projection of u_{1E} in the normal direction at the end-effector. The first and second links are commanded to rotate 10 degrees each; the controller gains obtained from Figure 5-2 are used for all four structures including the initial structure. Figures 5-6 and 5-7 show the time response of the original system, which has $D1=0.18\text{m}$ and $D2=0.18\text{m}$. Figure 5-6 is plotted with the rigid mode response; the solid line is for θ_1 and the dotted line is for θ_2 in Figure 5-6 (a). The time response of the joints shows the coupling effects of the two links where the second link is rotating backward at the beginning in Figure 5-6 (a). Figure 5-6 (b) is plotted in x-y coordinates where the initial point is (0,0) and the circle with cross is the target position. The overshoot for the rigid mode performance in Equation (5-9) is calculated from this plot. Figure 5-7 (a) shows the deflections at the end of each links with the solid line for u_{1E} and the dotted line for u_{2E} . Figure 5-7 (b) shows the resulting deflection at the

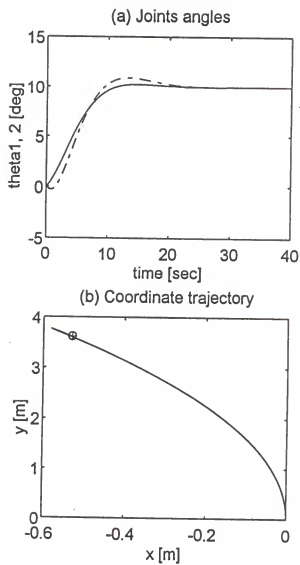


Figure 5-6 Rigid mode time response of the structure (D_1 and $D_2=0.18\text{m}$)

(a) Joint angles (θ_1 : solid line, θ_2 : dotted line)

(b) Coordinate trajectory of end point

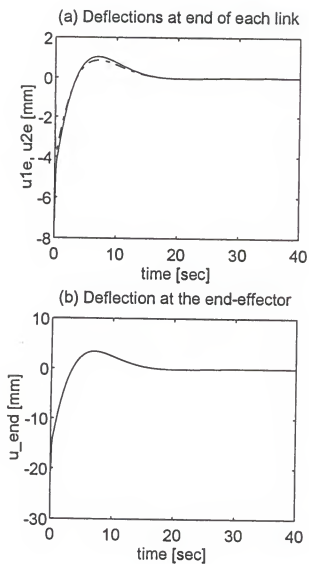


Figure 5-7 Flexible mode time response of the structure ($D1, D2=0.18\text{m}$)

(a) deflections at the end of each link

(b) deflection at the end-effector

end-effector as obtained by using Equation (5-10). The maximum value of the deflection from this plot is the measure of performance of the flexible mode in Equation (5-10).

Three new optimized systems with controller effects are also investigated with the same controller and the same step input command. The results are compared in Table 5-1 by using the overshoot of the rigid mode and the maximum deflection of the flexible mode. The values in parentheses are the percent difference in responses between the initial structure and the structure designed with consideration of the controller effects.

Table 5-1 System performances of the optimized systems with clamped free model

Controller	D1(m)	D2(m)	OS(%)	$u_{end}(mm)$	% avg.
	0.18	0.18	4.3997	20.6	0
fn_rl	0.17	0.15	4.0162(-8.72%)	27.8(34.9%)	13.1
kq_rl	0.16	0.15	4.0188(-8.66%)	32.6(58.3%)	24.8
mqr1*kqr1	0.19	0.15	4.0190(-8.65%)	21.0(1.9%)	-3.4

All structural designs incorporating controller effects decreased the overshoot of the rigid mode about 8.7 percent so that the performance of the rigid mode is improved. The flexible mode response showed worse response due to increased vibration. The third scheme, which used the

product of the modal mass and modal stiffness, showed the smallest vibration increase. The percent average change of the rigid and flexible modes are listed in last column as a total performance index based on equal importance of the two modes. Based on the percent average, the last design shows better performance compared to the initial structure. This table implies that the impulse response provides the best approach for modifying a given structural design.

The optimization using the product of the modal mass and modal stiffness is investigated further by changing the scale factor SF_3 in Equation (5-8). The resulting optimal points are plotted in Figure 5-8 with the scale factor SF_3 . When the scale factor decreases, the effects of the rigid mode controller increase so that the diameter of the first link is increased and that of the second link is decreased, as illustrated Figure 5-8. When SF_3 is 0.2, the resulting structure has $D1=0.24m$ and $D2=0.13m$; this structure showed better response as shown in Table 5-2. The system performance is improved by approximately 13 percent in both modes. This shows that careful consideration should be given to the selection of the scale factor in Equation 5-8.

Further study was undertaken for the clamped mass model with the initial structure $D1=0.18m$ and $D2=0.16m$ shown in Figure 4-8. All procedures to include the controller effects are exactly the same as those followed for the clamped free model. The controller gains are calculated

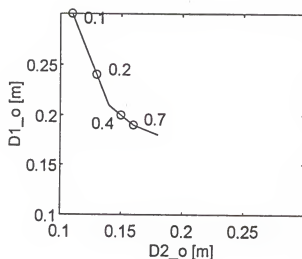


Figure 5-8 Effect of scale factor on optimal system with clamped free model
(The results are shown for $SF_3 = 0.1, 0.2, 0.4$ and 0.7 respectively)

Table 5-2 System performances of the optimized systems with clamped free model

Controller	D1(m)	D2(m)	OS(%)	$u_{end}(mm)$	% avg.
	0.18	0.18	4.3997	20.6	0
$m q_{r1} * k q_{r1}$	0.24	0.13	3.8453(-12.6%)	18.0(-12.6%)	-12.6

based on Equations (5-2) and (5-3). The proportional gains are $K_{p1}=4100$ and $K_{p2}=800$. The derivative gains are $K_{v1}=20500$ and $K_{v2}=4000$. The controller effects on the design stage were found to be best when the product of the modal mass and modal stiffness is included in the design stage by using U_8 in Equation (5-8). The scale factor SF_3 was 0.4008 based on Equation (4-4) and the resulting optimal structure was $D1=0.19m$

and $D2=0.14\text{m}$. The performance of this structure is compared to the initial system in Table 5-3 with 10 degree step inputs on each joint. The effects of the scale factor were investigated further. The optimal diameters plotted in Figure 5-9 are obtained by changing the scale factor SF_3 . This plot shows the same trend as in the clamped free model. Decreasing the scale factor increases the effect of the rigid mode and causes the diameter of the first link to increase and that of the second link to decrease. The optimal structure is chosen with the scale factor SF_3 0.3 from Figure 5-9 with $D1=0.20\text{m}$ and $D2=0.13\text{m}$. This system shows an improved response as given in Table 5-3. This again suggests that the scale factor should be chosen carefully. The resulting system shows higher improvement in both the rigid and flexible modes through the structural modification incorporating controller effects.

The optimal structure of the clamped free model has diameters of 0.23m and 0.13m for the first and second links respectively. Another optimal structure of the clamped mass model has $D1=0.2\text{m}$ and $D2=0.13\text{m}$ so that two different models show different optimal diameters for the first link. One reason for this is the dependence of the optimal structure on the initial structure. The optimal relationship between $D1$ and $D2$ in Figure 5-9 is controlled by changing the scale factor, which prevents evaluating any structure diverging from that optimal relationship line. The performance of the structure of the clamped mass model can be improved further by

increasing the diameter of the first link while maintaining the size of the second link, diverging from the optimal ratio derived from structural criteria.

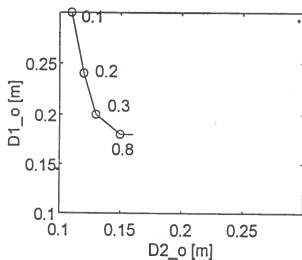


Figure 5-9 Effect of scale factor on optimal system with clamped mass model
(The results are shown for $SF_3 = 0.1, 0.2, 0.3$ and 0.8 respectively)

Table 5-3 System performances of the optimized systems with clamped mass model

Controller	D1(m)	D2(m)	OS(%)	$u_{end}(mm)$	% avg.
	0.18	0.16	4.1470	25.9	0
$mq_{r1} * kq_{r1}$	0.19	0.14	3.9233(-5.4%)	24.5(-5.4%)	-5.4
$mq_{r1} * kq_{r1}$	0.20	0.13	3.8290(-7.7%)	21.9(-15.4%)	-11.6

Conclusion

This chapter proposed a design methodology that includes controller effects in the design stage. This study shows the assumed mode method easily includes the effects of the controller. Two possible ways of including the controller in design optimization are through time or frequency domain analysis. The frequency domain approach is used in this study because it generates more general system information than the time domain approach. The time domain approach may show better results through more effective search algorithms but will provide less physical insight into the system's dynamic characteristics.

A simple PD-controller was found to be an effective tool for including controller effects in the design stage by taking into account both the rigid and flexible modes. The optimization schemes used were based on the natural frequency, the modal stiffness, and the product of the modal mass and modal stiffness in the rigid mode. All resulting structures showed better rigid mode dynamics by 5-13 percent due to effects of the controller. Different levels of penalties on the flexible mode depended on the scheme used. The optimization employing impulse response showed improved flexible mode response when appropriate scale factors were used.

The optimization scheme using the product of the modal mass and modal stiffness showed the best performance with scale factor 0.2-0.3, depending on the assumptions used in beam modeling. Emphasis on the effects of the controller decreased the diameter of the second link and

diminished flexible mode characteristics. Therefore, an optimal scale factor should be chosen carefully for improved flexible mode performance. This study shows that a larger first link and a smaller second link decreased overshoot in rigid mode dynamics. Flexible mode performance, however, showed strong dependence on the mass distribution of the two links.

CHAPTER 6 SUMMARY, CONCLUSION, AND RECOMMENDATION

Summary

The reported integrated design approach can be summarized in the following way:

Step 1: Structural design (clamped free model)

Given the link lengths L_1 , L_2 , t/D (thickness/diameter) and diameter constraints

Find the diameters D_1 and D_2 which minimize the following objective function

$$U_1 = \text{norm}(M) + SF_1 \times \left(\text{norm}\left(\frac{1}{fn\_f}\right) + SF_2 * \text{norm}\left(\frac{1}{fn\_rf}\right) \right)$$

where 'norm' = normalization of a matrix with respect to its maximum value

SF_1 , SF_2 = the scale factor based on Equation (4-4)

fn_f = the first flexible mode natural frequency of the manipulator
with joint-locked as a clamped structure

fn_rf = the first flexible mode natural frequency of the manipulator
with joint-unlocked as a pinned structure

Step 2: Proportional controller gain design

Given the optimal diameters D_1 and D_2 from step 1

Find the optimal proportional controller gains K_{p1} and K_{p2} in a selected domain of K_{p1} and K_{p2} to minimize the following objective function

$$U_4 = \text{norm}\left(\frac{1}{f_{n\_r1}}\right) + WF_1 \times \text{norm}\left(1 - \frac{f_{n\_r2}}{f_{n\_r2\_d}}\right)$$

where WF_1 = the weighting factor

f_{n_r1} = the first rigid mode natural frequency

f_{n_r2} = the second rigid mode natural frequency

$f_{n\_r2\_d}$ = one half of the first flexible mode natural frequency

with joint-locked based on $D1$ and $D2$, which is selected as a desired objective for the second rigid mode natural frequency.

Step 3: Derivative controller gain design

Given the optimal f_{n_r1} and f_{n_r2} corresponding to the optimal values of $D1$, $D2$, K_{p1} , and K_{p2}

Find the derivative controller gains K_{v1} and K_{v2} which minimize the following objective function

$$U_5 = \text{norm}\left(1 - \frac{\zeta_1}{0.707}\right) + WF_2 \times \text{norm}(\text{off-diag of } [Cq])$$

where ζ_1 = the modal damping for the first rigid body mode corresponding to the optimal values of $D1$, $D2$, K_{p1} , and K_{p2}

0.707 = the desired modal damping ratio.

off-diag of $[Cq]$ = off-diagonal terms of the modal damping matrix

Step 4: Including the controller effects in the structural design (with consideration of the rigid mode natural frequency)

Given the optimal values of Kp1 and Kp2 determined from step 2

Find the design values of D1 and D2 that minimize the following objective function U_6 by including the rigid mode controller effects on the structural design, U_1 from Step 1

$$U_6 = \text{norm}\left(\frac{1}{f_{n\_r1}}\right) + SF_3 \times \text{norm}\left(\text{norm}(M) + SF_1 \times \left(\text{norm}\left(\frac{1}{f_{n\_f}}\right) + SF_2 * \text{norm}\left(\frac{1}{f_{n\_rf}}\right)\right)\right)$$

where f_{n_r1} = the first rigid mode natural frequency with fixed Kp1 and Kp2 by changing D1 and D2

SF_3 = the scale factor based on Equation (4-4)

Step 5: Including the controller effects (with different objective functions)

Step 4 is repeated using the first rigid mode modal stiffness instead of the first rigid mode natural frequency. The objective function to be minimized is therefore,

$$U_7 = \text{norm}\left(\frac{1}{k_{q\_r1}}\right) + SF_3 \times \text{norm}\left(\text{norm}(M) + SF_1 \times \left(\text{norm}\left(\frac{1}{f_{n\_f}}\right) + SF_2 * \text{norm}\left(\frac{1}{f_{n\_rf}}\right)\right)\right)$$

where k_{q_r1} = the modal stiffness of the first rigid mode

Step 4 is also repeated using the product of the modal mass and the modal stiffness from the first rigid mode instead of the first rigid mode natural frequency. The objective function to be minimized in this case is

$$U_8 = \text{norm}(m_{q\_r1} \times k_{q\_r1}) + SF_3 \times \text{norm}\left(\text{norm}(M) + SF_1 \times \left(\text{norm}\left(\frac{1}{f_{n\_f}}\right) + SF_2 * \text{norm}\left(\frac{1}{f_{n\_rf}}\right)\right)\right)$$

where m_{q_r1} is the modal mass of the first rigid mode

Step 6: Comparison of the results

The initial optimum structure from Step 1 and the three optimized systems from Step 4 and Step 5 are compared through the time domain response of the manipulator to a step input at both joints. The rigid and flexible mode responses are compared with respect to minimization of vibration. The results showed that the objective function U_8 gives the best design in this respect.

Step 7: Effect of the scale factor

Changing the scale factor SF_3 in U_8 from 0.5671 to 0.2 achieves better results in the time domain response to a step input due to the change of the design diameters from $D1=0.19\text{m}$ and $D2=0.15$ to $D1=0.24\text{m}$ and $D2=0.13\text{m}$. This demonstrates that careful consideration should be given to the scale factor selection procedure.

Step 8: Effect of different model (clamped mass model)

The beam model is changed from the clamped free model to the clamped mass model. Steps 1-3 are repeated to find the initial structure and the controller gains. The initial structure in this case has the same size for the first link, but the diameter of the second link is decreased from 0.18m to 0.16m. When the controller effects are considered by using U_8 in Step 5, which includes the product of the modal mass and modal stiffness, the optimal dimensions changed from $D1=0.18\text{m}$ and $D2=0.16\text{m}$ to $D1=0.19\text{m}$ and $D2=0.14\text{m}$. Further improvement also was obtained when the scale factor SF_3 was changed from 0.4 to 0.2; the resulting structure

has $D1=0.20\text{m}$ and $D2=0.13\text{m}$. This suggests that the value of SF_3 in U_8 should be changed by trial and error or by a systematic procedure in order to obtain the best results in the time domain response.

Step 9: Effect of the objective function

The effect of the objective function on the structural design is investigated. Step 1 is repeated with two other objective functions as follows:

Case 1: Structural design (clamped free model, joint-free case)

(1-a) Using modal stiffness

The objective function is formulated using the modal stiffness of the first flexible mode with general torque inputs on both joints.

$$U_2 = \text{norm}(M) + SF_1 \times \left(\text{norm}\left(\frac{1}{kq}\right) \Big|_{w/T1} + SF_2 * \text{norm}\left(\frac{1}{kq}\right) \Big|_{w/T2} \right)$$

where $T1$ = the general torque input at the first joint

$T2$ = the general torque input at the second joint.

(1-b) Using the product of the modal mass and the modal stiffness

The objective function is formulated using the product of the modal mass and modal stiffness of the first flexible mode with general torque inputs on both joints.

$$U_3 = \text{norm}(M) + SF_1 \times \left(\text{norm}\left(\frac{1}{mq \times kq}\right) \Big|_{w/T1} + SF_2 * \text{norm}\left(\frac{1}{mq \times kq}\right) \Big|_{w/T2} \right)$$

The difference between U_2 and U_3 is incorporating the modal mass in the objective function. Including the modal mass with torque inputs at the

joints changed the optimal structure from $D1=0.18\text{m}$ and $D2=0.18\text{m}$ based on U_2 to $D1=0.18\text{m}$ and $D2=0.16\text{m}$ based on U_3 .

Case 2: Structural design (clamped free model, joint-locked case)

(2-a) Using modal stiffness

The objective function is formulated using the modal stiffness of the first flexible mode with general disturbance inputs on the end of each link. It is expressed as:

$$U_2 = \text{norm}(M) + SF_1 \times \left(\text{norm}\left(\frac{1}{kq}\right) \Big|_{w/d1} + SF_2 * \text{norm}\left(\frac{1}{kq}\right) \Big|_{w/d2} \right)$$

where $d1$ = the general disturbance at the end of the first link

$d2$ = the general disturbance at the end of the second link.

(2-b) Using the product of the modal mass and the modal stiffness

The objective function in this case is formulated as follows using the product of the modal mass and the modal stiffness of the first flexible mode with general disturbance inputs on the end of each link.

$$U_3 = \text{norm}(M) + SF_1 \times \left(\text{norm}\left(\frac{1}{mq \times kq}\right) \Big|_{w/d1} + SF_2 * \text{norm}\left(\frac{1}{mq \times kq}\right) \Big|_{w/d2} \right)$$

The difference between U_2 and U_3 is incorporating the modal mass. Including the modal mass with general disturbance inputs increased the diameter of the first link and decreased the diameter of the second link from $D1=0.17\text{m}$ and $D2=0.15\text{m}$ based on U_2 to $D1=0.19\text{m}$ and $D2=0.14\text{m}$ based on U_3 .

Conclusion

The following can be concluded from the reported study based on the investigated cases:

1. Dominant flexible modes and beam modeling: The clamped free model and the clamped mass model showed almost the same dominant flexible mode when the joints are locked. When the joints are unlocked, the dominant flexible modes from the two models gave different results and the clamped mass model showed a much lower natural frequency in the flexible mode.
2. Effect of beam modeling on the optimal line in the design domain: The lines giving the optimal relation of the two link diameters were different for the clamped free and clamped mass models. The clamped mass model showed a larger difference between the diameters of the first and second links.
3. Effect of the general joint inputs on the optimal line: The optimal lines showed different results for the different joint inputs. The optimal lines indicate that the link subjected to an input should be bigger than the other.
4. Effect of the general disturbances at the link ends: The effect of the general disturbances at the ends of the links can be evaluated by combination of the two disturbances on the first flexible mode at the end of each link.

5. Modal mass of the flexible mode: The inclusion of the modal mass in the objective function increased the diameter of the first link and decreased the diameter of the second link for the same mass. The results were the same in the joint-locked and joint-free cases.
6. Optimization based on the natural frequency and the impulse response: When the joints are locked, the optimal lines for the relation between the two link diameters were almost the same when the natural frequency or the product of the modal mass and modal stiffness were used in the objective function.
7. Optimization based on the natural frequency and the modal stiffness: When the joints are locked, the two optimization criteria based on the natural frequency and the modal stiffness produced different optimal lines. Using the natural frequency increased the difference between the first and the second link diameters.
8. Controller design: The pole assignments were performed to find the controller gains by using the control theory and the optimization theory. The optimization theory was used to make the first rigid mode frequency as high as possible and to uncouple the modal equations of the rigid mode.
9. Controller and rigid mode parameters: The virtual structural parameters for the rigid mode were calculated by using the PD controller. Consequently, the resulting system has mass, damping, and stiffness for both the rigid and flexible modes.

10. Optimization with the controller: The controller is included in the design stage to modify the structure based on the natural frequency, the modal stiffness, and the product of the modal mass and modal stiffness of the rigid mode. The product of the modal mass and modal stiffness produced best results with respect to reducing the vibration by increasing the first link diameter and decreasing the second link diameter. The resulting system showed improved dynamic characteristics for both the rigid and flexible modes.

11. Effect of the rigid mode modal mass: The modal mass of the rigid mode has not previously received much attention in the design stage. This study shows that the modal mass plays an important role in both the rigid and flexible mode responses.

12. Search algorithm: When the controller is included in the design stage, the optimal parameters of the structure are determined by following the optimal lines in the design domain and changing the scale factors. This is an inherent limitation of the search method used to find a better structure. Accordingly, it is expected that a search algorithm which explores the entire design space would produce better solutions.

Recommendation

The followings are recommendations for future investigation.

1. Problem solving approach: Although the proposed design procedure for the structure with controller effects produced good results, it is not

necessarily the best way to design a manipulator with optimized performance since the controller design is based on the frequency domain analysis. A time domain design approach with fewer constraints and searching the entire design space for the optimum design is recommended for future work in order to guarantee the best performance for the integrated system.

2. Configuration and task effects: The effects of the change of manipulator structural configuration and incorporating the specified tasks in the optimization procedure should be considered for developing robust system design.

3. Condition at the end-effector: The effects of additional mass and applied forces at the end-effector can be important factors for practical robust system design. This should also be incorporated in the optimum design procedure.

4. Condition at the joints: The controller at the joints can change the dynamic characteristics of the mechanical system. The flexibility of the joints also can significantly influence the dynamics of the system and should be considered in future study.

5. Trade-off analysis: Trade-off between the different design objectives plays an important role in weighing the effects of two conflicting properties by the scale factor selection. More practical knowledge on the relative merits of the conflicting properties should be incorporated for the robust design of real world manipulators performing specific functions.

APPENDIX A SYSTEM ENERGY EQUATIONS

A.1 Kinetic energy equation

$$\begin{aligned}
 T = & \frac{1}{2}(J_h + J_0 + m_j L_1^2) \dot{\theta}_1^2 + \frac{1}{2} m_1 (\dot{q}_{11}^2 + \dot{q}_{12}^2) + \frac{1}{2} m_j (\phi_{11E} \dot{q}_{11} + \phi_{12E} \dot{q}_{12})^2 \\
 & + \dot{\theta}_1 \left[(nw_{11} + m_j L_1 \phi_{11E}) \dot{q}_{11} + (nw_{12} + m_j L_1 \phi_{12E}) \dot{q}_{12} \right] \\
 & + \frac{1}{2} (m_2 + m_p) L_1^2 \dot{\theta}_1^2 + \frac{1}{2} (m_2 + m_p) (\phi_{11E} \dot{q}_{11} + \phi_{12E} \dot{q}_{12})^2 \\
 & + \frac{1}{2} (m_2 + m_p) \dot{\theta}_1^2 (\phi_{11E} q_{11} + \phi_{12E} q_{12})^2 \\
 & + \frac{1}{2} (J_{01} + J_p) (\dot{\theta}_1 + \dot{\theta}_2 + \dot{u}'_{1E})^2 + \frac{1}{2} m_2 (\dot{q}_{21}^2 + \dot{q}_{22}^2) + \frac{1}{2} m_p (\phi_{21E} \dot{q}_{21} + \phi_{22E} \dot{q}_{22})^2 \\
 & + (m_2 + m_p) L_1 \dot{\theta}_1 (\phi_{11E} \dot{q}_{11} + \phi_{12E} \dot{q}_{12}) + (m_2 + 2m_p) \frac{L_1 L_2}{2} \dot{\theta}_1 (\dot{\theta}_1 + \dot{\theta}_2 + \dot{u}'_{1E}) \cos \theta_2 \\
 & + L_1 \dot{\theta}_1 \cos \theta_2 \left[(m_p \phi_{21E} + nq_{21}) \dot{q}_{21} + (m_p \phi_{22E} + nq_{22}) \dot{q}_{22} \right] \\
 & - L_1 \dot{\theta}_1 (\dot{\theta}_1 + \dot{\theta}_2 + \dot{u}'_{1E}) \left[(m_p \phi_{21E} + nq_{21}) q_{21} + (m_p \phi_{22E} + nq_{22}) q_{22} \right] \sin \theta_2 \\
 & + (\phi_{11E} \dot{q}_{11} + \phi_{12E} \dot{q}_{12}) (\dot{\theta}_1 + \dot{\theta}_2 + \dot{u}'_{1E}) (m_2 + 2m_p) \frac{L_2}{2} \cos \theta_2 \\
 & + (\phi_{11E} \dot{q}_{11} + \phi_{12E} \dot{q}_{12}) \left[(m_p \phi_{21E} + nq_{21}) \dot{q}_{21} + (m_p \phi_{22E} + nq_{22}) \dot{q}_{22} \right] \cos \theta_2 \\
 & - (\phi_{11E} \dot{q}_{11} + \phi_{12E} \dot{q}_{12}) (\dot{\theta}_1 + \dot{\theta}_2 + \dot{u}'_{1E}) \left[(m_p \phi_{21E} + nq_{21}) q_{21} + (m_p \phi_{22E} + nq_{22}) q_{22} \right] \sin \theta_2 \\
 & + (m_2 + 2m_p) (\phi_{11E} q_{11} + \phi_{12E} q_{12}) \dot{\theta}_1 (\dot{\theta}_1 + \dot{\theta}_2 + \dot{u}'_{1E}) \frac{L_2}{2} \sin \theta_2 \\
 & + (\phi_{11E} q_{11} + \phi_{12E} q_{12}) \dot{\theta}_1 \left[(m_p \phi_{21E} + nq_{21}) \dot{q}_{21} + (m_p \phi_{22E} + nq_{22}) \dot{q}_{22} \right] \sin \theta_2 \\
 & + (\phi_{11E} q_{11} + \phi_{12E} q_{12}) \dot{\theta}_1 (\dot{\theta}_1 + \dot{\theta}_2 + \dot{u}'_{1E}) \left[(m_p \phi_{21E} + nq_{21}) q_{21} + (m_p \phi_{22E} + nq_{22}) q_{22} \right] \cos \theta_2 \\
 & + (\dot{\theta}_1 + \dot{\theta}_2 + \dot{u}'_{1E}) \left[(nw_{21} + m_p L_2 \phi_{21E}) \dot{q}_{21} + (nw_{22} + m_p L_2 \phi_{22E}) \dot{q}_{22} \right] \\
 & + \frac{1}{2} J_p (\dot{\theta}_1 + \dot{\theta}_2 + \dot{u}'_{1E} + \dot{u}'_{2E})^2
 \end{aligned}$$

where

$$J_0 = \int_{m_1} x_1^2 dm = \frac{m_1 L_1^2}{3} + \frac{m_1 (R_{1,i}^2 + R_{1,o}^2)}{4}$$

$$J_{01} = \int_{m_2} x_2^2 dm + m_p (L_2)^2 = \frac{m_2 L_2^2}{3} + \frac{m_2 (R_{2,i}^2 + R_{2,o}^2)}{4} + m_p (L_2)^2$$

$$\phi_{ijR} = \phi_{ij} \text{ at the end of the link}$$

$$nq_{ij} = \int_0^l \bar{m}_i \phi_{ij}(x) dx = \bar{m}_i \frac{2a_j}{\lambda_j} = \frac{2a_j}{L_j} m_i$$

$$(a_1 = 0.7341, a_2 = 1.0185 \text{ for clamped free model})$$

$$nw_{ij} = \int_0^l \bar{m}_i x \phi_{ij}(x) dx = \bar{m}_i \frac{2}{\lambda_j^2} = \frac{2m_i l}{L_j^2} \quad (L_j = \lambda_j l)$$

A.2 Potential energy equation

$$\begin{aligned} V &= \frac{1}{2} \int_0^{L_1} EI_1 \left(\frac{\partial^2 u_1}{\partial x_1^2} \right)^2 dx + \frac{1}{2} \int_0^{L_2} EI_2 \left(\frac{\partial^2 u_2}{\partial x_2^2} \right)^2 dx \\ &= \frac{EI_1}{2} \lambda_1^4 L_1 q_{11}^2 + \frac{EI_1}{2} \lambda_2^4 L_1 q_{12}^2 + \frac{EI_2}{2} \lambda_1^4 L_2 q_{21}^2 + \frac{EI_2}{2} \lambda_2^4 L_2 q_{22}^2 \end{aligned}$$

APPENDIX B SYSTEM EQUATIONS

These are the linearized system equations utilized in Chapter 3 and the components of the mass, damping, stiffness, and torque coefficient matrices are listed.

$$\begin{aligned}
 M_{11}\ddot{\theta}_1 + M_{12}\ddot{\theta}_2 + M_{13}\ddot{q}_{11} + M_{14}\ddot{q}_{12} + M_{15}\ddot{q}_{21} + M_{16}\ddot{q}_{22} + b_1\dot{\theta}_1 &= T_1 \\
 M_{21}\ddot{\theta}_1 + M_{22}\ddot{\theta}_2 + M_{23}\ddot{q}_{11} + M_{24}\ddot{q}_{12} + M_{25}\ddot{q}_{21} + M_{26}\ddot{q}_{22} + b_2\dot{\theta}_2 &= T_2 \\
 M_{31}\ddot{\theta}_1 + M_{32}\ddot{\theta}_2 + M_{33}\ddot{q}_{11} + M_{34}\ddot{q}_{12} + M_{35}\ddot{q}_{21} + M_{36}\ddot{q}_{22} + C_1\dot{q}_{11} + K_1q_{11} &= T_2\phi'_{11E} + F_1\phi_{11E} \\
 M_{41}\ddot{\theta}_1 + M_{42}\ddot{\theta}_2 + M_{43}\ddot{q}_{11} + M_{44}\ddot{q}_{12} + M_{45}\ddot{q}_{21} + M_{46}\ddot{q}_{22} + C_2\dot{q}_{12} + K_2q_{12} &= T_2\phi'_{12E} + F_1\phi_{12E} \\
 M_{51}\ddot{\theta}_1 + M_{52}\ddot{\theta}_2 + M_{53}\ddot{q}_{11} + M_{54}\ddot{q}_{12} + M_{55}\ddot{q}_{21} + M_{56}\ddot{q}_{22} + C_3\dot{q}_{21} + K_3q_{21} &= F_2\phi_{21E} \\
 M_{61}\ddot{\theta}_1 + M_{62}\ddot{\theta}_2 + M_{63}\ddot{q}_{11} + M_{64}\ddot{q}_{12} + M_{65}\ddot{q}_{21} + M_{66}\ddot{q}_{22} + C_4\dot{q}_{22} + K_4q_{22} &= F_2\phi_{22E}
 \end{aligned}$$

where

$$M_{ij} = M_{ji} \quad i, j = 1, 2, \dots, 6$$

$$M_{11} = (J_h + J_o + m_j L_1^2) + (m_2 + m_p) L_1^2 + (J_{o1} + J_p) + (m_2 + 2m_p) L_1 L_2$$

$$M_{12} = (J_{o1} + J_p) + (m_2 + 2m_p) \frac{L_1 L_2}{2}$$

$$\begin{aligned}
 M_{13} = & (nw_{11} + m_j L_1 \phi_{11E}) + (m_2 + m_p) L_1 \phi_{11E} + (\phi_{11E} + L_1 \phi'_{11E})(m_2 + 2m_p) \frac{L_2}{2} \\
 & + (J_{h2} + J_{o2} + J_p + m_p L_2^2) \phi'_{11E}
 \end{aligned}$$

$$M_{14} = (nw_{12} + m_j L_1 \phi_{12E}) + (m_2 + m_p) L_1 \phi_{12E} + \phi_{12E} (m_2 + 2m_p) \frac{L_2}{2}$$

$$M_{15} = L_1 m_p \dot{p}_{21} + (nw_{21} + m_p L_2 \phi_{21E})$$

$$M_{16} = L_1 m_p \dot{p}_{22} + (nw_{22} + m_p L_2 \phi_{22E})$$

$$M_{22} = (J_{o1} + J_p)$$

$$M_{23} = \phi_{11E}(m_2 + 2m_p) \frac{L_2}{2}$$

$$M_{24} = \phi_{12E}(m_2 + 2m_p) \frac{L_2}{2}$$

$$M_{25} = (nw_{21} + m_p L_2 \phi_{21E})$$

$$M_{26} = (nw_{22} + m_p L_2 \phi_{22E})$$

$$M_{33} = m_1 + (m_j + m_2 + m_p) \phi_{11E}^2 + (J_{h2} + J_{o2} + J_p + m_p L_2^2) \phi_{11E}'^2$$

$$M_{34} = (m_j + m_2 + m_p) \phi_{11E} \phi_{12E} + (J_{h2} + J_{o2} + J_p + m_p L_2^2) \phi_{11E}' \phi_{12E}'$$

$$M_{35} = \phi_{11E} m p_{21}$$

$$M_{36} = \phi_{11E} m p_{22}$$

$$M_{44} = m_1 + (m_j + m_2 + m_p) \phi_{12E}^2 + (J_{h2} + J_{o2} + J_p + m_p L_2^2) \phi_{12E}'^2$$

$$M_{45} = \phi_{12E} m p_{21}$$

$$M_{46} = \phi_{12E} m p_{22}$$

$$M_{55} = m_2 + m_p \phi_{21E}^2 + J_p \phi_{21E}'^2$$

$$M_{56} = m_p \phi_{21E} \phi_{22E} + J_p \phi_{21E}' \phi_{22E}'$$

$$M_{66} = m_2 + m_p \phi_{22E}^2 + J_p \phi_{22E}'^2$$

$$K_{ij} = EI_i \int_0^{L_i} (\phi_{ij}''(x))^2 dx = EI_i \mathcal{A}_i^4 L_i \quad (K_1 = K_{11}, K_2 = K_{12}, K_3 = K_{21}, K_4 = K_{22})$$

$$M_{ij} = \int_0^{L_i} \bar{m}_i (\phi_{ij}(x))^2 dx = \bar{m}_i L_i = m_i \quad (M_1 = M_{11}, M_2 = M_{12}, M_3 = M_{21}, M_4 = M_{22})$$

$$b_n = 0$$

$$C_n = 2\zeta \frac{K_{\text{eff}}}{W_n} = 2\zeta \sqrt{K_n M_n}$$

$$TC_{11} = 1, \quad TC_{22} = 1, \quad (\text{off-diag of } [TC] = 0)$$

$$TC_{32} = \phi'_{11E}, \quad TC_{42} = \phi'_{12E}$$

$$TC_{33} = \phi_{11E}, \quad TC_{43} = \phi_{12E}$$

$$TC_{54} = \phi_{21E}, \quad TC_{64} = \phi_{22E}$$

APPENDIX C FINITE ELEMENT PROGRAM

This is the finite element program to check the structural characteristics of the two link manipulator. The session file of COSMOS is listed for the example in the Chapter 2.

```
PLANE,Z,0,1,
SCALE,0,
VIEW,0,0,1,0,
MPROP,1,DENS,1744,EX,145.3E9,EY,145.3E9,
CRPCORD,1,0,10,0,7,10,0,7,10,0,
CRPCORD,2,7,10,0,14,10,0,14,10,0,
EGROUP,1,BEAM2D,0,0,0,0,0,0,0,
BMSECD,1,1,3,1,7,0.19,0.19*0.02,0,0,0,0,0,
M_CR,1,1,1,2,7,1,
BMSECD,1,2,3,1,7,0.15,0.15*0.02,0,0,0,0,0,
M_CR,2,2,1,2,7,1,
EGROUP,2,MASS,0,0,0,0,0,0,0,
RCONST,2,3,1,7,50,50,0,0,0,0.1,0,
M_PT,2,2,1,
RCONST,2,4,1,7,100,100,0,0,0,100,0,
M_PT,3,3,1,
DND,1,AL,0,1,1,
NMERGE,1,18,1,0.0001,0,0,0,
NCOMPRESS,1,16,
A_FREQ,4,S,16,0,0,0,0,1E-005,0,1E-006,0,0,0,0,
R_FREQ,
FREQLIST;
```

APPENDIX D GENERAL PROGRAM LISTINGS

This appendix includes programs for controller design, system parameters, mode shapes, mass matrix, and stiffness matrix.

D.1 This sub-program is to find the proportional gains based on the Equation (5-2).

```

Diameter=[D1_o D2_o];
sys_par; mode_shp; m_linear; k_f_line;
fn=sort(sqrt(eig(inv(M)*K))/2/pi)';
fn_f=sort(sqrt(eig(inv(M(3:6,3:6))*K(3:6,3:6)))/2/pi)';
fn_r_max=min(fn_f)*0.5;
n=0;div=100;
Kp1_min=100;Kp1_max=10000;d_Kp1=(Kp1_max)/div;
Kp2_min=100;Kp2_max=10000;d_Kp2=(Kp1_max)/div;
for Kp1=Kp1_min:d_Kp1:Kp1_max,
n=n+1; m=0;
    for Kp2=Kp2_min:d_Kp2:Kp2_max,
        m=m+1;
        K(1,1)=Kp1; K(2,2)=Kp2;
        fn_r=sort(sqrt(eig(inv(M(1:2,1:2))*K(1:2,1:2)))/2/pi)';
        fn1=fn_r(1);fn2=fn_r(2);
        obj1(n,m)=1/fn1;
        obj2(n,m)=abs(1-fn2/fn_r_max);
    end
end
obj11=obj1/max(max(obj1));obj22=obj2/max(max(obj2));
U4=obj11+obj22;U4=U4/max(max(U4));

```

D.2 This sub-program is to find the derivative gains based on the Equation (5-3).

```

K(1,1)=Kp1;K(2,2)=Kp2;
n=0;
Kv1_min=100;Kv1_max=40000; d_Kv1=(Kp1_max)/div;
Kv2_min=100;Kv2_max=40000;d_Kv2=(Kp1_max)/div;
for Kv1=Kv1_min:d_Kv1:Kv1_max,
n=n+1; m=0;
    for Kv2=Kv2_min:d_Kv2:Kv2_max,
        m=m+1;

```

```

C(1,1)=Kv1; C(2,2)=Kv2;
fn_r=sort(sqrt(eig(inv(M(1:2,1:2))*K(1:2,1:2)))/2/pi);
M_r=M(1:2,1:2); K_r=K(1:2,1:2); C_r=C(1:2,1:2);
[v_r,d_r]=eig(inv(M_r)*K_r);
diag_d=sort(diag(d_r));
for nn=1:2,
    if (d_r(nn,nn)~= diag_d(1))
        x_r(:,1)=v_r(:,nn);
    end
    if (d_r(nn,nn)~= diag_d(2))
        x_r(:,2)=v_r(:,nn);
    end
end
x_r(:,1)=x_r(:,1)/x_r(1,1); x_r(:,2)=x_r(:,2)/x_r(1,2);
p_r=x_r;
mq_r=diag(p_r'*M_r*p_r);
cq_r=diag(p_r'*C_r*p_r);
kq_r=diag(p_r'*K_r*p_r);
zeta1=cq_r(1)/(2*sqrt(mq_r(1)*kq_r(1)));
zeta2=cq_r(2)/(2*sqrt(mq_r(2)*kq_r(2)));
Cq_mat=p_r'*C_r*p_r;
obj4(n,m)=abs(Cq_mat(1,2));
obj5(n,m)=abs(1-zeta1/0.707);
end
end
obj44=obj4/max(max(obj4)); obj55=obj5/max(max(obj5));
U5=obj44+obj55; U5=U5/max(max(U5));

```

D.3 This sub-program is to calculate the system parameters.

```

gr=9.80665; Ratio1=0.02; Ratio2=0.02;
thick1=D1_o*Ratio1; thick2=D2_o*Ratio2;
D1_i=D1_o-2*thick1; D2_i=D2_o-2*thick2;
R1_o=D1_o/2.; R1_i=D1_i/2.; R2_o=D2_o/2.; R2_i=D2_i/2.;
A1=pi*(R1_o^2-R1_i^2); A2=pi*(R2_o^2-R2_i^2);
L1=7.; L2=7.;
Rho=1744; E=145.3e9;
I1=pi/64*(D1_o^4-D1_i^4); I2=pi/64*(D2_o^4-D2_i^4);
m1=Rho*A1*L1; m2=Rho*A2*L2; mh2=50;
mp=100; Jp=100;
Jh1=0.1; Jh2=0.1;
JL1=m1/4.*(R1_i^2+R1_o^2)+m1*L1^2/3.;
JL2=m2/4.*(R2_i^2+R2_o^2)+m2*L2^2/3.;

```

D.4 This sub-program is to calculate the system mode shape based on Equation (3-13) and (3-14).

```

clmp_mass=1; % 1 for clamped mass model, 0 for clamped free model
%First link
mass_r1=(mh2+m2+mp)/m1;
num=1; freq_eq_old=1;
for B_l=0.01:0.01:5
    no_payload=1+cos(B_l)*cosh(B_l);
    w_payload=mass_r1*B_l*(cos(B_l)*sinh(B_l)-cosh(B_l)*sin(B_l));
    freq_eq=no_payload+w_payload*clmp_mass;
    if(freq_eq*freq_eq_old) < 0
        B_l1(num)=B_l;
        num=num+1;
    end
    freq_eq_old=freq_eq;
    if num == 3
        break
    end
end
BB1=B_l1(1); BB2=B_l1(2);
%BB1=1.875104; BB2=4.694091; %This is for the clamped free model.
SS1=(cosh(BB1)+cos(BB1))/(sinh(BB1)+sin(BB1));
SS2=(cosh(BB2)+cos(BB2))/(sinh(BB2)+sin(BB2));
%Second link
mass_r2=mp/m2;
num=1; freq_eq_old=1;
for B_l=0.01:0.01:5
    no_payload=1+cos(B_l)*cosh(B_l);
    w_payload=mass_r2*B_l*(cos(B_l)*sinh(B_l)-cosh(B_l)*sin(B_l));
    freq_eq=no_payload+w_payload*clmp_mass;
    if(freq_eq*freq_eq_old) < 0
        B_l1(num)=B_l;
        num=num+1;
    end
    freq_eq_old=freq_eq;
    if num == 3
        break
    end
end
BB3=B_l1(1); BB4=B_l1(2);
%BB3=BB1; BB4=BB2;; %This is for the clamped free model.
SS3=(cosh(BB3)+cos(BB3))/(sinh(BB3)+sin(BB3));
SS4=(cosh(BB4)+cos(BB4))/(sinh(BB4)+sin(BB4));

```

```

Phi_11E=(cosh(BB1)-cos(BB1))-SS1*(sinh(BB1)-sin(BB1));
Phi_12E=(cosh(BB2)-cos(BB2))-SS2*(sinh(BB2)-sin(BB2));
Phi_21E=(cosh(BB3)-cos(BB3))-SS3*(sinh(BB3)-sin(BB3));
Phi_22E=(cosh(BB4)-cos(BB4))-SS4*(sinh(BB4)-sin(BB4));
Phi_11E_P=BB1/L1*(sinh(BB1)+sin(BB1))-SS1*(cosh(BB1)-cos(BB1));
Phi_12E_P=BB2/L1*(sinh(BB2)+sin(BB2))-SS2*(cosh(BB2)-cos(BB2));
Phi_21E_P=BB3/L2*(sinh(BB3)+sin(BB3))-SS3*(cosh(BB3)-cos(BB3));
Phi_22E_P=BB4/L2*(sinh(BB4)+sin(BB4))-SS4*(cosh(BB4)-cos(BB4));

```

D.5 This sub-program is to calculate the system mass matrix based on Appendix B.

%This part is for the clamped free model.

```

nq11=2*SS1/BB1*m1; nq12=2*SS2/BB2*m1;
nq21=2*SS3/BB3*m2; nq22=2*SS4/BB4*m2;
nw11=2/BB1^2*L1*m1; nw12=2/BB2^2*L1*m1;
nw21=2/BB3^2*L2*m2; nw22=2/BB4^2*L2*m2;

```

%This part is for the clamped mass model.

```

C1=BB1/L1;C2=BB2/L1;C3=BB3/L1;C4=BB4/L1;ddxx=0.01;
x=0; ddx: 7;

```

```

phi_x1=(cosh(C1*x)-cos(C1*x))-SS1*(sinh(C1*x)-sin(C1*x));
phi_x2=(cosh(C2*x)-cos(C2*x))-SS2*(sinh(C2*x)-sin(C2*x));
phi_x3=(cosh(C3*x)-cos(C3*x))-SS3*(sinh(C3*x)-sin(C3*x));
phi_x4=(cosh(C4*x)-cos(C4*x))-SS4*(sinh(C4*x)-sin(C4*x));
summ1=phi_x1.*x*ddxx;summ11=sum(summ1);
summ2=phi_x2.*x*ddxx;summ21=sum(summ2);
summ3=phi_x3.*x*ddxx;summ31=sum(summ3);
summ4=phi_x4.*x*ddxx;summ41=sum(summ4);
summ5=phi_x1*ddxx;summ51=sum(summ5);
summ6=phi_x2*ddxx;summ61=sum(summ6);
summ7=phi_x3*ddxx;summ71=sum(summ7);
summ8=phi_x4*ddxx;summ81=sum(summ8);
nw11=m1/L1*summ11;nw12=m1/L1*summ21;
nw21=m2/L2*summ31;nw22=m2/L2*summ41;
nq11=m1/L1*summ51;nq12=m1/L1*summ61;
nq21=m2/L2*summ71;nq22=m2/L2*summ81;

```

```

mp21=mp*Phi_21E+nq21;mp22=mp*Phi_22E+nq22;cos2is1=1.;
M(1,1)= Jh1+JL1+mh2*L1^2+(m2+mp)*L1^2+ JL2+mp*L2^2+Jp
        +(m2+2*mp)*L1*L2*cos2is1;
M(1,2)= Jh2+JL2+Jp+mp*L2^2+(m2+2*mp)*L1*L2/2.*cos2is1 ;

```

```

M(1,3)= (nw11+mh2*L1*Phi_11E)+(m2+mp)*L1*Phi_11E+Phi_11E
        *(m2+2*mp)*L2/2*cos2is1+(Jh2+JL2+Jp+mp*L2^2)*Phi_11E_P
        +Phi_11E_P*L1*(m2+2*mp)*L2/2*cos2is1;

```

$$M(1,4) = (nw12 + mh2 * L1 * \Phi_{12E}) + (m2 + mp) * L1 * \Phi_{12E} + \Phi_{12E} \\ * (m2 + 2 * mp) * L2 / 2 * \cos 2is1 + (Jh2 + JL2 + Jp + mp * L2^2) * \Phi_{12E\_P} \\ + \Phi_{12E\_P} * L1 * (m2 + 2 * mp) * L2 / 2 * \cos 2is1;$$

$$M(1,5) = L1 * mp21 * \cos 2is1 + (nw21 + mp * L2 * \Phi_{21E}) + Jp * \Phi_{21E\_P};$$

$$M(1,6) = L1 * mp22 * \cos 2is1 + (nw22 + mp * L2 * \Phi_{22E}) + Jp * \Phi_{22E\_P};$$

$$M(2,1) = M(1,2);$$

$$M(2,2) = Jh2 + JL2 + Jp + mp * L2^2;$$

$$M(2,3) = \Phi_{11E} * (m2 + 2 * mp) * L2 / 2 * \cos 2is1 + (Jh2 + JL2 + Jp + mp * L2^2) * \Phi_{11E\_P};$$

$$M(2,4) = \Phi_{12E} * (m2 + 2 * mp) * L2 / 2 * \cos 2is1 + (Jh2 + JL2 + Jp + mp * L2^2) * \Phi_{12E\_P}$$

$$M(2,5) = (nw21 + mp * L2 * \Phi_{21E}) + Jp * \Phi_{21E\_P};$$

$$M(2,6) = (nw22 + mp * L2 * \Phi_{22E}) + Jp * \Phi_{22E\_P};$$

$$M(3,1) = M(1,3); \quad M(3,2) = M(2,3);$$

$$M(3,3) = m1 + mh2 * \Phi_{11E}^2 + (m2 + mp) * \Phi_{11E}^2 + 2 * (m2 * L2 / 2 + mp * L2) \\ * \Phi_{11E} * \Phi_{11E\_P} * \cos 2is1 + (Jh2 + JL2 + Jp + mp * L2^2) * \Phi_{11E\_P}^2;$$

$$M(3,4) = mh2 * \Phi_{11E} * \Phi_{12E} + (m2 + mp) * \Phi_{11E} * \Phi_{12E} + (m2 * L2 / 2 \\ + mp * L2) * (\Phi_{11E} * \Phi_{12E\_P} + \Phi_{12E} * \Phi_{11E\_P}) * \cos 2is1 + (Jh2 + JL2 \\ + Jp + mp * L2^2) * \Phi_{11E\_P} * \Phi_{12E\_P};$$

$$M(3,4) = (m2 * L2 / 2 + mp * L2) * (\Phi_{11E} * \Phi_{12E\_P} + \Phi_{12E} * \Phi_{11E\_P}) \\ * \cos 2is1 + (Jh2 + JL2 + Jp + mp * L2^2) * \Phi_{11E\_P} * \Phi_{12E\_P}; \text{(CM model)}$$

$$M(3,5) = \Phi_{11E} * mp21 * \cos 2is1 + (nw21 + Jp * \Phi_{21E\_P} + mp * L2 * \Phi_{21E}) \\ * \Phi_{11E\_P};$$

$$M(3,6) = \Phi_{11E} * mp22 * \cos 2is1 + (nw22 + Jp * \Phi_{22E\_P} + mp * L2 * \Phi_{22E}) \\ * \Phi_{11E\_P};$$

$$M(4,1) = M(1,4); \quad M(4,2) = M(2,4); \quad M(4,3) = M(3,4);$$

$$M(4,4) = mh2 * \Phi_{12E} * \Phi_{12E} + m1 + (m2 + mp) * \Phi_{12E} * \Phi_{12E} + 2 * (m2 * L2 / 2 \\ + mp * L2) * \Phi_{12E} * \Phi_{12E\_P} * \cos 2is1 + (Jh2 + JL2 + Jp + mp * L2^2) \\ * \Phi_{12E\_P}^2;$$

$$M(4,5) = \Phi_{12E} * mp21 * \cos 2is1 + (nw21 + Jp * \Phi_{21E\_P} + mp * L2 * \Phi_{21E}) \\ * \Phi_{12E\_P};$$

$$M(4,6) = \Phi_{12E} * mp22 * \cos 2is1 + (nw22 + Jp * \Phi_{22E\_P} + mp * L2 * \Phi_{22E}) \\ * \Phi_{12E\_P};$$

$$M(5,1) = M(1,5); M(5,2) = M(2,5); M(5,3) = M(3,5); M(5,4) = M(4,5);$$

$$M(5,5) = m2 + mp * \Phi_{21E}^2 + Jp * \Phi_{21E\_P}^2;$$

$$M(5,6) = mp * \Phi_{21E} * \Phi_{22E} + Jp * \Phi_{21E\_P} * \Phi_{22E\_P};$$

$$M(5,6) = Jp * \Phi_{21E\_P} * \Phi_{22E\_P}; \text{(CM model)}$$

$$M(6,1) = M(1,6); M(6,2) = M(2,6); M(6,3) = M(3,6); M(6,4) = M(4,6); M(6,5) = M(5,6);$$

$$M(6,6) = m2 + mp * \Phi_{22E}^2 + Jp * \Phi_{22E\_P}^2;$$

D.6 This sub-program is to calculate the system stiffness matrix based on Appendix B.
 $K=zeros(6,6);$

%This part is stiffness matrix elements for the clamped free model.

```

EI1=E*I1; EI2=E*I2;
K(1,1)=0;K(1,2)=0;K(2,1)=0; K(2,2)=0;
K(3,3)=1.875104^4*EI1/L1^3;
K(4,4)=4.694091^4*EI1/L1^3;
K(5,5)=1.875104^4*EI2/L2^3;
K(6,6)=4.694091^4*EI2/L2^3;

```

%This part is stiffness matrix elements for the clamped mass model.

```

C1=BB1/L1;C2=BB2/L1;C3=BB3/L1;C4=BB4/L1;ddxx=0.01;
x=0:ddxx:7;
phi_ddx1=((cosh(C1*x)+cos(C1*x))-SS1*(sinh(C1*x)+sin(C1*x)))*C1^2;
phi_ddx2=((cosh(C2*x)+cos(C2*x))-SS2*(sinh(C2*x)+sin(C2*x)))*C2^2;
phi_ddx3=((cosh(C3*x)+cos(C3*x))-SS3*(sinh(C3*x)+sin(C3*x)))*C3^2;
phi_ddx4=((cosh(C4*x)+cos(C4*x))-SS4*(sinh(C4*x)+sin(C4*x)))*C4^2;
summ_K3=(phi_ddx1.^2)*ddxx; summ_K33=sum(summ_K3);
summ_K4=(phi_ddx2.^2)*ddxx; summ_K44=sum(summ_K4);
summ_K5=(phi_ddx3.^2)*ddxx; summ_K55=sum(summ_K5);
summ_K6=(phi_ddx4.^2)*ddxx; summ_K66=sum(summ_K6);
K(3,3)=summ_K33*EI1;
K(4,4)=summ_K44*EI1;
K(5,5)=summ_K55*EI2;
K(6,6)=summ_K66*EI2;

```

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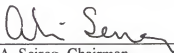
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